

UNITED STATES DISTRICT COURT EASTERN DISTRICT OF NEW YORK

UNITED STATES OF AMERICA)	11-CR-414 (JBW)
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-against-)	
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LAWRENCE DICRISTINA)	
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Defendant.)	
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REPORT OF RANDAL D. HEEB, PHD

July 5, 2012

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I. Introduction

I.A. Qualifications and experience

- (1) I am a Partner in the economic consulting firm of Bates White, LLC, where I am a co-leader of the firm's Intellectual Property and Antitrust business practices. I have a Bachelor of Arts degree in Economics from the University of Washington, a Master in Public Administration degree from the Harvard Kennedy School, and a PhD in Economics from the University of Chicago. I have held various academic posts, most recently as a Senior Faculty Fellow at the Yale School of Management. I have also held positions as an Assistant Professor of Economics at INSEAD, a graduate business school located near Paris, France, a visiting Assistant Professor of Strategy at the University of Chicago Booth School of Business, and a Lecturer in Economics at the University of Chicago.
- (2) In the past, I have been retained to testify as an expert economist by both plaintiffs and defendants in litigation, and I have been qualified as an expert to provide opinions to the court based on econometric analyses. I have also been retained by both plaintiffs and defendants to lead teams of economists to conduct analyses and prepare expert reports in support of other testifying experts and to assess damages and liability in intellectual property, antitrust, breach of contract, fraud, and other business disputes. I have presented economic analyses, including game theoretical and econometric analyses, to the staffs and economists of the US Federal Trade Commission, several States Attorneys General and the Competition Bureau of Canada on multiple occasions on topics related to competition in a variety of industries.
- (3) In my capacity as a Senior Faculty Fellow at the Yale School of Management, I have taught MBA students in both core and advanced MBA strategy courses, lecturing on topics including techniques in game theory and the application of game theory to business problems. Throughout those courses, I employ examples from poker to illustrate various skills, techniques, and lessons that contribute to success in both business and poker. At INSEAD, I designed and taught a popular MBA elective strategy course based entirely on these game theory lessons. I have also taught game theory applications to PhD students in industrial organization courses at INSEAD, to undergraduates at the University of Chicago, and to business executives in graduate training programs. I have published scholarly work involving the application of game theoretical concepts to competitive situations.
- (4) I have also been trained in econometrics, the branch of mathematics and statistics involving the application of statistical techniques to economic data. I have written and published scholarly work employing econometric techniques to distinguish the impacts of individual choices from the effects of other confounding factors. My position as a consulting economist routinely involves work in econometrics.

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- (5) I have been a successful non-professional poker player for more than 25 years, playing live and more recently online in both “cash games” and tournaments. In 1994, I reached the final table in a major Nevada poker tournament for the first time, playing Limit Hold’em. I have won or finished in the money in U.S. or International poker tournaments in games of Limit Hold’em, No Limit Hold’em, Pot Limit Omaha, and Omaha Hi/Lo. I have finished in the money in World Series of Poker events six times, including 2002, when I won the World Series of Poker \$3,000 No Limit Hold’em event.

I.B. Scope of charge

- (6) I have been asked to evaluate whether skill predominates over chance in poker. In reaching my opinions, I have considered my own experience playing poker and teaching others the game of poker, my own experience and expertise in economics, game theory, and econometrics, as well as a variety of publicly available materials and non-public materials. With respect to non-public materials, I analyzed approximately 415 million poker hands played on the online poker site PokerStars from April 2010 through March 2011.
- (7) Bates White LLC is being compensated at a rate of \$650 per hour for my time working on this matter. I have been assisted in my work by a team of economists and consultants from Bates White working under my direct supervision. Bates White is being compensated for their work at their customary bill rates. Neither my own nor Bates White’s compensation is dependent upon the conclusions that I have reached or the outcome of this case.

I.C. Summary of opinions

- (8) Based upon my experience and expertise, and my review and analysis of approximately 415 million hands played on the online poker site PokerStars, I conclude that skill predominates over chance in poker. I base this conclusion on several opinions and analyses, which are described in detail in this report and summarized below.
- (9) First, poker is a game that involves an enormous number of complex decisions, and there are numerous skills that players can develop to influence the outcome of a poker game. These complex strategic considerations allow players of varying skill to differentiate themselves in poker.
- (10) Second, many people make a living playing poker and win consistently over time. This fact illustrates a key difference between poker and games in which chance predominates over skill. A game in which a skillful player can reliably earn a living is a game in which skill predominates over chance. Poker is such a game, and this fact alone is an independent foundation for my opinion that skill predominates over chance in poker.

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- (11) Third, using the data I received from PokerStars, I analyzed whether a player's performance – defined as dollars won per hand played – with one specific hand is related to that player's performance with a completely different set of hands. I find that players who earn more when dealt one group of starting hands consistently earn more when dealt other starting hands, and these results are statistically significant.¹ In other words, successful players consistently win more than less successful players with (virtually) all starting hands. This result is to be expected if poker is a game in which skill predominates over chance. The skills, techniques, and strategies that make players successful with one hand are the same skills that make them successful with other hands. Conversely, if poker were a game of chance, there would be no relationship between a player's outcome with one hand and a player's outcome with another hand. These relationships are statistically significant and persist across various different hands and stakes. These results provide strong evidence that players' skill determines poker outcomes and that skill predominates over chance in determining outcomes in poker.
- (12) Fourth, using the data I received from PokerStars, I implemented statistical analyses to estimate the skill level of players based upon a large number of variables regarding the strategies and tactics they employ. If poker were a game of chance, skill would be a non-factor and any attempt at predicting a player's skill level, and therefore that player's likelihood of success, would be futile. I find that a player's skill level is in fact a good predictor of that player's outcomes (i.e., how much money they win per hand played). The fact that players who are predicted to be more skillful do in fact have better results than players of lesser predicted skill provides independent corroboration for my opinion that skill predominates over chance in determining outcomes in poker.
- (13) Fifth, using data I received from PokerStars and my skill estimates, I used "Monte Carlo" simulation techniques to test whether more skillful players predominate over less skillful players. I approach this question by asking: "If a higher skilled player and a lower skilled player both played the same number of hands, how many hands would it take before the more skillful player is at least 90% likely to be ahead of (i.e., to have won more money or lost less money than) the less skillful player?" The precise answer depends on the stakes chosen, but in each of the stakes, more skillful players predominate over less skillful players after a number of hands that can be played in a few sessions of poker. Based on this result, and independently of the other analyses I conducted upon which I reached the same conclusion, it is my opinion that skill predominates over chance in poker.
- (14) Sixth, because I have identified several independent tests, each of which leads me separately to the conclusion that poker is a game predominated by skill, my confidence in this conclusion is extremely high. Each of these tests would be sufficient by itself to establish that poker is predominately a game

¹ Statisticians and economists use the phrase "statistically significant" to describe a result that has a probability of having occurred by chance of less than a particular threshold. Unless I specify otherwise, throughout this report I will use "statistically significant" to mean, "statistically significant at the 95% confidence level," implying that the result is less than 5% likely to have occurred by chance.

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of skill. The fact that multiple independent tests all lead to the same conclusion demonstrates that this result is very robust.

- (15) I conducted my analyses on No Limit Hold'em online cash games played on 6-player maximum tables at stakes from \$0.50/\$1.00 through \$10/\$20. Based on my experience as a poker player and as an econometrician, it is my opinion that this result, that poker is a game in which skill predominates over chance, extends directly to other closely related poker games. In particular, this result extends to poker games played on 9 and 10 player max tables. It also extends to all stakes, however much higher or lower than those that I analyzed. It also extends to poker games played with Limit and Pot-Limit betting rules. It extends to Omaha Hold'em and Omaha Hi/Lo Hold'em. It also extends to tournament play. It extends to all combinations and permutations of the above. A complete discussion of the extensions of these conclusions to closely related games can be found in Section IV.G of the Appendix.
- (16) I performed my empirical analyses on data drawn from online poker games. Online poker differs from live poker in one important respect - live poker allows scope for even greater contribution from skills related to players' abilities to "read" their opponents, and thereby deduce the cards that their opponents are likely to hold from their opponents' demeanor and behavior. This additional skill factor that is present in live poker but largely absent from online poker means that live poker is at least as much of a game of skill as online poker. All of the conclusions I have drawn above from my analyses of online poker are equally if not more true for live poker.

II. Background

II.A. Elements of both skill and chance exist in nearly all games and competitions

- (17) It is important to recognize that nearly all competitions involve elements of both skill and chance. Thus, the question of whether a particular game is purely skill or purely chance is ill-conceived, as essentially all games have elements of both. Consider, at one end of the continuum, games that primarily involve chance. For example, calling heads or tails on a coin flip is pure chance. Similarly, roulette is a game of pure chance – the player chooses a number and hopes for the best. On the other end of the continuum are games that are commonly thought of as games of skill. These games include professional sports or chess, for example. While outcomes in these games are primarily determined by skill, chance still plays a role in these games. For example, consider the role of a "bad bounce" in baseball or golf or the role of varying weather conditions in football. While skill clearly plays a central role, chance also undeniably has some influence in determining outcomes in these games.

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- (18) The aforementioned points are important to keep in mind as we evaluate the role of skill versus chance in poker. In particular, like most games and competitions, both skill and chance play a role in poker. Thus, the key question is not whether chance plays some role, since it plays a role in almost all games and competitions, but rather whether skill predominates over chance in determining outcomes in poker.

II.B. Poker involves numerous complex decisions

- (19) Poker is a game that involves an enormous number of complex decisions. While it would not be possible to describe all such decisions and considerations here, below I provide an overview of several critical questions that arise in a typical poker hand. This discussion demonstrates the vastly complex nature of poker and the many skills that players use to influence the outcome. In Section III.E, I conduct several statistical analyses that incorporate many of the concepts discussed in this section.
- (20) When a poker hand begins, each player receives her personal cards (“hole cards”) from the dealer.² The players act one after another in clockwise order. Each player to act must first choose whether to discard their hand (“fold”), to meet the minimum bet required for the table (“call”), or to increase the amount required to play the hand (“raise”). This decision alone, which is one of potentially dozens of decisions that will be made throughout each hand, should be the result of analyzing a large amount of information already available at a player’s first decision.
- (21) When deciding whether or not to play a hand, the first question one usually asks is, “what are my hole cards?” There are some hands that should almost never be played. There are other hands that should almost always be played. But for the vast majority of starting hands, the decision depends on the answers to a much more complex set of questions.
- (22) If an earlier player has elected to play the hand, one should generally be more selective because, by betting, the earlier player has suggested she holds a strong hand. For example, a hand such as an Ace and a Jack of different suits (e.g., A♣ J♦), which is a hand with positive expected profits for most players when played as the first player to enter the pot, has less value when another player has entered the pot earlier. This is due to the fact that the player entering the pot earlier may have a hand that is likely to win against A♣ J♦, such as A♣ K♣ or A♣ Q♣.
- (23) Further, one must consider the caliber of the players who have already entered the pot. It can be profitable to play a less-desirable hand against a weaker player.

² For expositional ease, I will use the feminine pronouns when referring to other players throughout this report.

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- (24) One also must interpret the earlier player's actions based on past experience with that player. Does this player raise more frequently than would a skillful player, indicating that the group of starting hands she is willing to play includes some weaker hands? Conversely, does she play less often than usual, and rarely start a hand outside of the highest ranking hands? When this player gets a strong hand, does she typically call with that hand, trying to lure other players into the pot, or does she raise to try to win what is already in the pot?
- (25) Additionally, one should consider things like how much money this player has at the table ("stack size"), and therefore, how much money one stands to win. Certain hands are more profitable when stack sizes are large, and other hands are more profitable when stack sizes are small. As money trades hands throughout the course of a game, the appeal of certain hands against certain players is constantly changing.
- (26) One should also consider the attributes and skill of remaining players who have not acted yet. If there are very aggressive and/or talented players who have yet to act, the appeal of playing a marginal hand decreases. The order of the players at the table ("position") is very important. Poker is a game of gathering and analyzing information, so the more information one has available when making decisions, the more likely it is those decisions will be profitable. Therefore, acting as late as possible in the hand is preferred, because players acting later will have seen the decisions of all the other players in the hand ahead. Because of this informational advantage, players acting later in the round can afford to play a wider variety of hands.
- (27) There is a great deal of skill involved in deducing the cards one's opponents hold from their decisions. One important element of strategy involves making bets that have the effect of forcing an opponent to reveal information. Thus, for example, it might make sense to raise a pot with a hand that is not quite strong enough, on its own, to justify the added money if the raise causes an opponent to reveal (by calling or raising) how strong her hand is. This added information may allow a skillful player to save more money on subsequent rounds if the player reads her opponent as having a strong hand, or to win the pot with a bluff later in the hand if she perceives weakness.
- (28) When deciding when and how to play, one must avoid becoming too predictable. Therefore it is desirable to mix in some element of randomization, and avoid playing exactly the same way each time, even in identical circumstances. Thus, one's strategy should evolve in response to an opponent who appears to be learning from the pattern of one's previous play.
- (29) All of the above considerations and more are weighed carefully by a skillful player before making even the first decision in a given hand: whether to fold, raise, or call on the first betting round, which is called "before the flop" or "pre-flop," for reasons that will become clear momentarily. The first betting round continues until all players have called all previous bets or folded. There are often several pre-flop decisions by a single player, involving a multitude of factors. However, I will forego

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further discussion of additional pre-flop decisions. My purpose in touching on some of these strategic elements is to illustrate just some of the ways that more skillful play differs from less skillful play. A description of each such pre-flop decision would span many pages. The statistical exercises that I discuss in later sections of this report include analyses informed by these observations about more and less skillful play before the flop.

- (30) After the first betting round, the dealer reveals the first three communal cards, which are called “the flop.” When the dealer reveals the flop, the players again act one after another in clockwise order, starting from the dealer’s left. The first player to act must re-evaluate the strength of her hand in light of the additional cards that are available to make the best five card poker hand. For example, if a player’s hole cards are an Ace and a Jack of a different suit ($A\clubsuit J\spadesuit$), and the flop is Ace, Seven, and Three of different suits ($A\heartsuit 7\heartsuit 3\clubsuit$), then the player now has improved to having a pair of Aces, with a Jack “kicker” (when two players make the same pair, the tie is broken by the strength of the second card, known as the kicker). Alternatively, if the flop is Ten, Seven, and Three of different suits ($10\spadesuit 7\heartsuit 3\clubsuit$), then the player has not improved her hand, and still holds only an Ace high.
- (31) The goal at this point continues to be to maximize profit (or minimize loss) from this situation. A fundamental question that arises is whether one has the best hand out of everyone who remains in the hand. The answer to this question requires an understanding of probability, as well as the tendencies of the opponents. With a top pair of Aces with a Jack kicker, one should ask “How frequently is that the best hand against the range of possible holdings of the particular opponents still in this pot?” Against most opponents, a pair of Aces with a good kicker would be favored, but against an opponent known to start only premium hands one must still be cautious, since she could have a better pair of Aces (e.g., an Ace and a Queen ($A\clubsuit Q\clubsuit$), instead of an Ace and a Jack ($A\clubsuit J\clubsuit$)). If there is the possibility to make a straight or a flush (five cards in order regardless of suit, or five of the same suit, respectively), one must consider how likely it is that an opponent has already made a straight or flush, or is lacking only one card to make such a hand. The ability to consistently interpret these possibilities and make profit maximizing decisions in each specific situation is one of the attributes that distinguishes more skillful players from less skillful players.
- (32) There are many additional elements of skillful play on the flop beyond the scope of what I discuss in detail here. For example, one must determine not only whether or not one is ahead, but how likely other players are to catch up when subsequent cards are dealt, or alternatively, how likely one is to catch up if an opponent in fact has the superior hand. The ability to calculate probabilities and associated betting odds is one of the more notable skills of a successful poker player. Such abilities guide a player in determining the proper amount to bet or deciding whether or not to call an opponent’s bet. Once again, the statistical exercises that I discuss in later sections of this report include analyses informed by these observations about the characteristics of more and less skillful play on the flop.

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- (33) The deal of the fourth common card (“the turn”) and fifth common card (“the river”) signal the beginning of the two final betting rounds. On each of these rounds, players must assess the strength of their hands and their opponents’ hands, and make decisions about how aggressively to bet, or whether to call or raise. At each stage and with each decision, players signal or reveal information about their own hands and elicit information from their opponents. Skillful play involves weighing many factors, such as stack sizes, pot sizes, the tendencies of opponents, the ability of opponents to recognize the tendencies of other players and their reactions to each other. With any given bet on any betting round, a skillful player may recognize that she does not have a good enough chance to win to justify continuing to play the hand, and the profit maximizing decision is to fold. Choosing correctly more often than does one’s opponents is what leads to profitable play. These are some of the elements of skillful poker strategy reflected, in part, in the analyses that follow.

III. Statistical analysis of poker hands confirms that skill predominates over chance

III.A. Description of available data

- (34) I received and analyzed over 415 million poker hands played on the online site PokerStars from the period April 2010 through March 2011. These poker hands covered all No Limit Texas Hold’em cash games denominated in U.S. dollars played on PokerStars during this timeframe ranging from stakes of \$0.50/\$1.00 blinds to \$10/\$20 blinds. For each hand played at these stakes, I was provided data identifying all substantive information about the hand, including but not limited to:
- The stakes of the game (e.g., \$1/\$2 blinds, \$5/\$10 blinds, etc.)
 - The game id (i.e., a unique identifier of every hand dealt)
 - The players’ screen names involved in the game
 - The players’ stack sizes (i.e., how much money they had in play)
 - The players’ hole cards (i.e., the two cards they were dealt)
 - The flop, turn, and river (if applicable)
 - The “action” on every street (i.e., whether each player checked, bet, raised, called, or folded at each point in the hand and how much money they called, bet, or raised)
 - Which player(s) won the hand
 - For hands that went to a “showdown,” the hand the winning player made (e.g., a straight, two pair, flush, etc.)

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- A record of any conversation that took place in the chat window during the hand
- Time stamps and other information that allow for the tracking of games and players through time

III.B. Validation of available data

- (35) To confirm the reliability of the data I received from PokerStars, I obtained publicly available data from HandHQ, a company that tracks hands played on PokerStars and other online poker sites. The HandHQ data can be thought of as a “bird’s eye” view from the perspective of an observer watching the game. That is, all information that would be available to an observer watching the game is tracked by HandHQ, but non-public information, such as every player’s hole cards, is not available. Data from HandHQ can be used by online poker players to analyze their own play as well as their opponents’ play. In particular, players can purchase a text file report of hands played from HandHQ and then analyze them to determine the tendencies or rate the skill of other players that they may encounter.
- (36) I used the HandHQ data to verify that the hands received from PokerStars were accurate records of hands played. Since the HandHQ records of actual hands played are an accepted source of data used by actual online poker players, they provide a good source to independently verify the information from PokerStars.
- (37) Without exception, for all hands for which I have a complete record from both PokerStars and HandHQ to be able to compare with one another, the information about the play of the hand is identical. On this basis, I conclude that the PokerStars data I received are a reliable and accurate source of information on actual hands played on the website. I have matched over 170 million hands from the HandHQ data to their corresponding hands from the PokerStars data. The vast majority, over 99.4% of the hands that I observed in the HandHQ data, was also observed in the PokerStars data. The missing hands (0.56% of all hands in the HandHQ data) all come from a few hours of play for which I do not have PokerStars data. The missing intervals occur during 56 of the 8,760 hours in the PokerStars database. In addition, a very small percentage of hands in the HandHQ data is truncated, and therefore cannot be matched with PokerStars hands. I have manually examined the text hand history files from HandHQ, and in all cases the problem appears in the underlying HandHQ data. Over 40% of the hands in the PokerStars data are also reflected and validated in the HandHQ data. This is consistent with my expectations, because the HandHQ data were represented to be a partial sample of all hands played on PokerStars during the sample period.

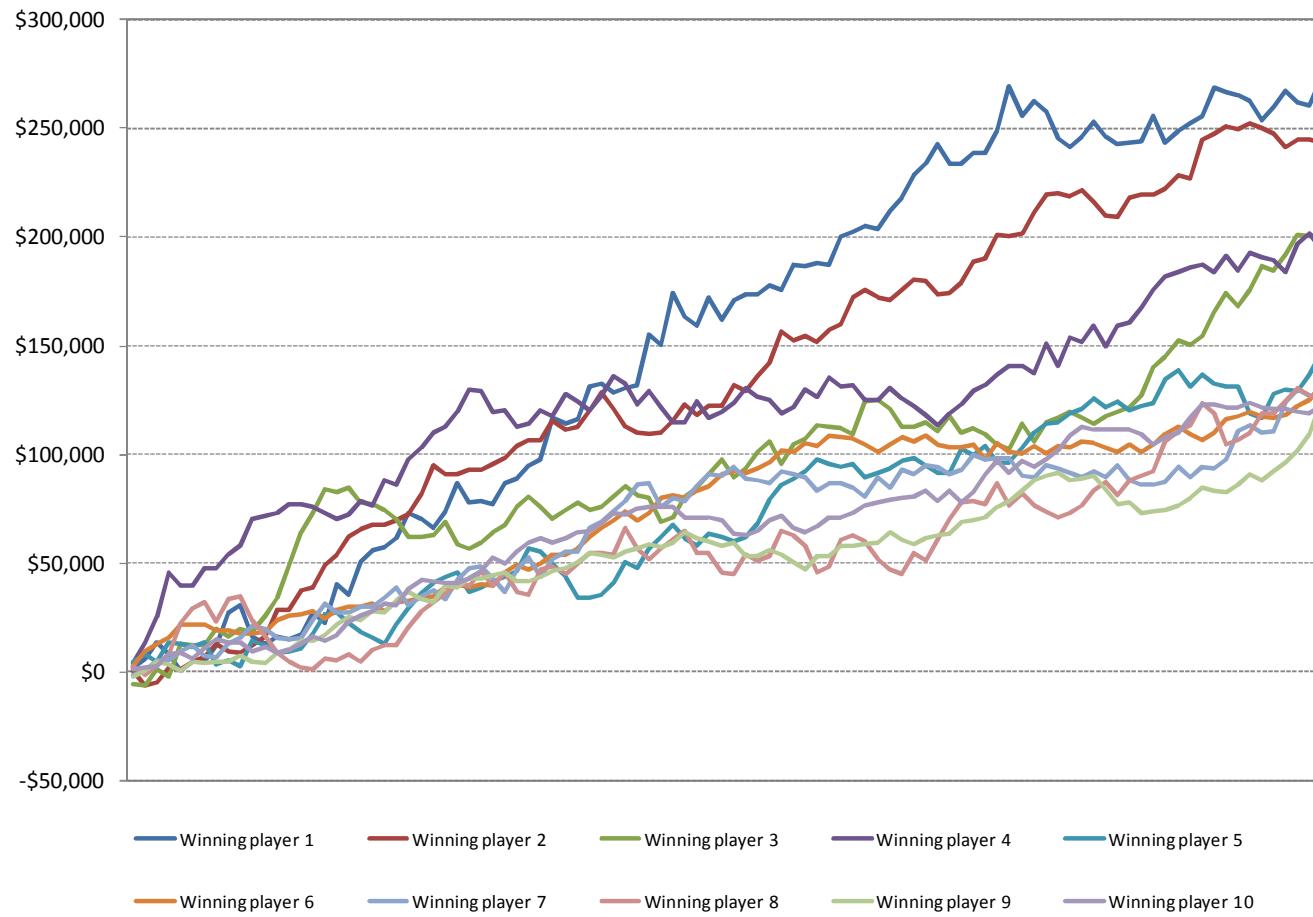
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III.C. Summary statistics

- (38) At the very outset, it is important to emphasize that many people make a living playing poker and win consistently over time. This fact illustrates a key difference between poker and all games in which chance predominates. For example, unlike poker, it is impossible to make a living and to win consistently playing casino games such as roulette. In contrast, poker is a game in which players can have a sufficient skill advantage to win consistently and make a living playing the game. This fact alone is an independent foundation for my opinion that skill predominates over chance in poker.
- (39) To illustrate the fact that players can and do win consistently and earn a living playing poker, I analyzed the top 10 players in terms of overall dollar amounts won during the year of data I received. Figure 1 below shows the cumulative amounts won during the year April 2010 through March 2011 for the top 10 players at \$5/\$10 stakes. Several features of the graph merit discussion. First, these players are clearly winning enough to earn a living playing poker. For example, the top player earned over \$250,000 playing \$5/\$10 stakes during the year. All of the top 10 players earned well in excess of \$100,000. In addition, these players win consistently. As the year progresses, these players' cumulative amounts won are nearly always increasing. Although they may have a few losing days or weeks, they tend to win consistently.

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Figure 1: Winnings through time (April 2010 through March 2011) for the top 10 players in terms of total dollar amounts won at \$5/\$10 stakes



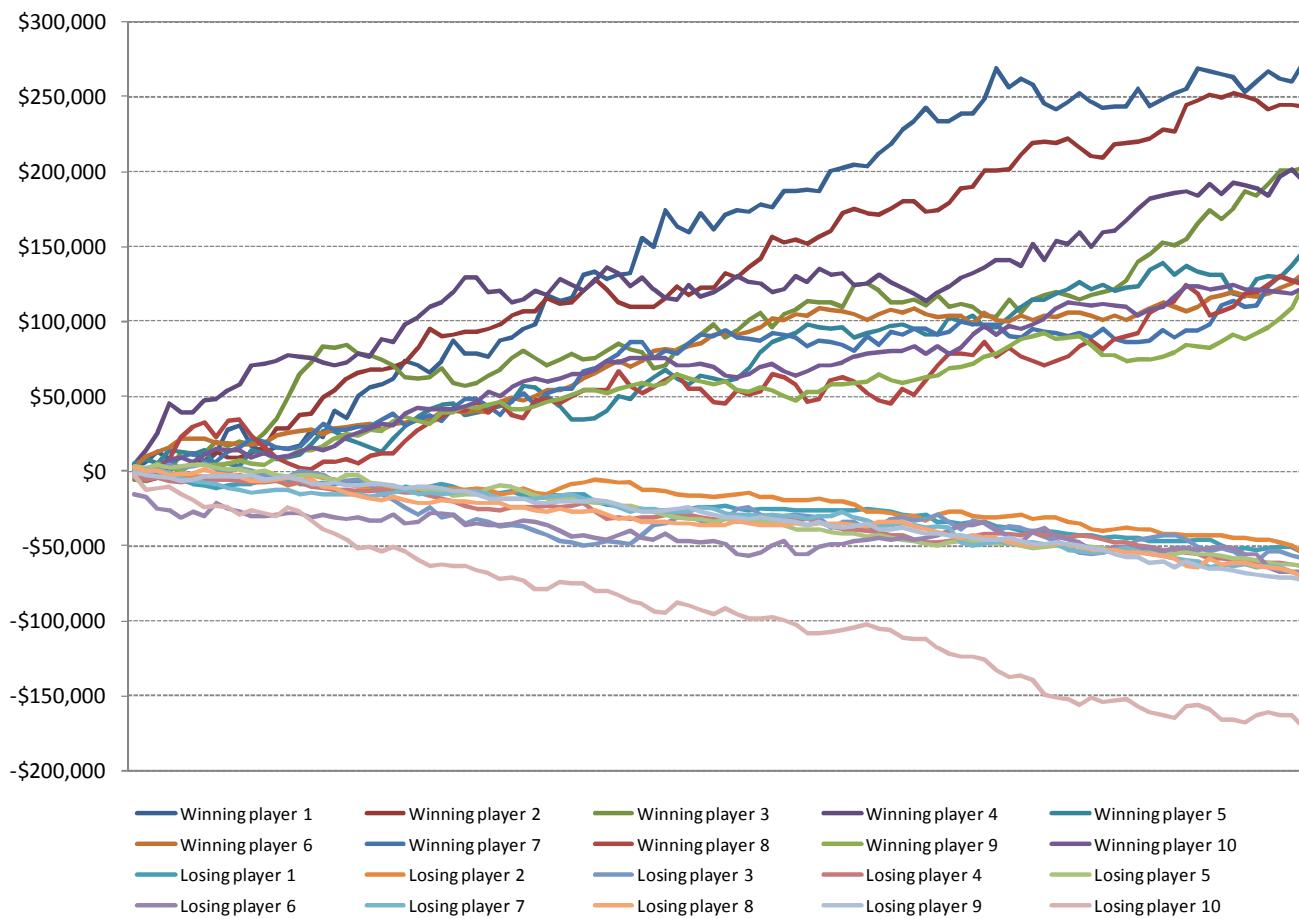
Note: For illustrative purposes, in this figure, all the hands played by each player are distributed evenly from left to right across the graph. That is, for example, the midpoint of the graph marks the midpoint of each player's playing history. Some players played more hands than others, and they may have played them at different times. However, all of the players depicted in the graph played a high volume of hands.

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- (40) It is also important to note that there are players that tend to lose consistently through time. In Figure 2, I provide the same chart shown above, but I add the cumulative results for the top 10 *losing* players at \$5/\$10 stakes. Notably, they are consistently losing throughout the year. The fact that the winning players tend to win consistently and the losing players tend to lose consistently demonstrates that there is a skill differential between these groups.

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Figure 2: Winnings through time (April 2010 through March 2011) for the top and bottom 10 players in terms of total dollar amounts won or lost at \$5/\$10 stakes



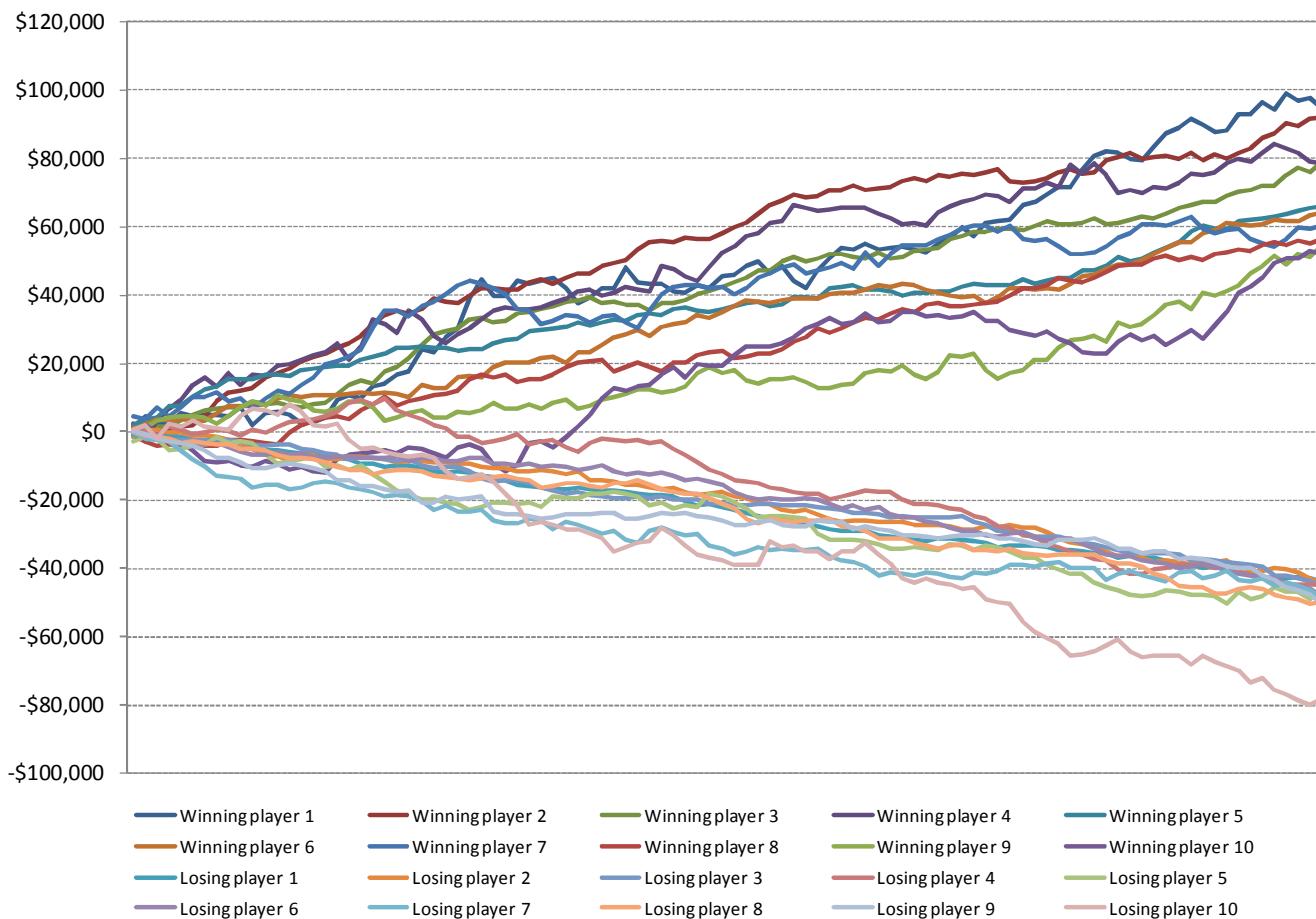
Note: For illustrative purposes, in this figure all the hands played by each player are distributed evenly from left to right across the graph. Thus, for example, the midpoint of the graph marks the midpoint of each player's playing history. Some players played more hands than others, and they may have played them at different times. However, all of the players depicted in the graph played a high volume of hands.

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- (41) Figure 3 below shows the same statistics for \$1/\$2 stakes. The results once again show players earn enough to make a living playing poker, even at \$1/\$2 stakes which are relatively modest stakes compared to \$5/\$10 stakes. In particular, the top 10 winning players all earn more than \$50,000. In addition, as we observed for \$5/\$10 stakes, the winning players tend to win consistently and the losing players tend to lose consistently, suggesting a persistent skill differential between these groups.

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Figure 3: Winnings through time (April 2010 through March 2011) for the top and bottom 10 players in terms of total dollar amounts won or lost at \$1/\$2 stakes



Note: For illustrative purposes, in this figure all the hands played by each player are distributed evenly from left to right across the graph. Thus, for example, the midpoint of the graph marks the midpoint of each player's playing history. Some players played more hands than others, and they may have played them at different times. However, all of the players depicted in the graph played a high volume of hands.

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- (42) Several other points merit discussion regarding the players observed in the data. Specifically, the top players tend to play a very large volume of hands and earn considerable money. For example, the top 50 winning players at \$1/\$2 stakes played 854,717 hands each, on average, and profited \$43,662 each, on average. In addition, consistent with the graphs shown above, the top 50 players tend to win consistently. Dividing the play of the top 50 players at \$1/\$2 stakes into individual monthly results, 439 of these months were winning months, and only 117 months were losing months. In contrast, dividing the play of the 50 largest *losing* players into individual monthly results, only 85 of these months were winning months, and 437 months were losing months. Again, the substantial and persistent disparities between the top winning and losing players demonstrate a persistent skill differential between the two groups. I find similar disparities for all of the stakes, as shown in the Appendix, Figure 17 of Section IV.A.

III.D. Illustrative analyses of winning persistence in poker

- (43) Using the data I received from PokerStars, I performed several analyses to evaluate the role of skill and chance in poker games. In the first set of analyses, I explore whether a player's performance – defined as dollars won per hand played – with one specific hand is related to her performance with a completely different set of hands. If poker were primarily a game of chance, there would be no reason to expect a player who performs well with one set of hands to also perform well with a different set of hands. Rather, results would be out of the player's control, driven by chance, and would not be consistent across different hands. In contrast, if skill drives outcomes in poker, then we would expect to see players that are successful with one specific hand also tend to be successful with a separate set of hands because the players' skills, techniques, and strategies – i.e., how well they play their hands – are driving their outcomes.
- (44) While there are a vast number of approaches one could take in evaluating this question, I start with a straightforward illustration based upon the hole cards that are dealt. In particular, there are 169 possible combinations of the two hole cards a player receives in Texas Hold'em. These include all pocket pairs (i.e., two of the same card, such as two Sixes (6♣ 6♦)), all suited combinations of non-paired cards (i.e., two cards of the same suit, such as an Eight of spades and a Ten of spades (8♣ 10♣)), and unsuited combinations of non-paired cards (i.e., two cards not of the same suit that are not paired, such as an Eight of spades and a Ten of diamonds (8♣ 10♦)). Note that, for each of these 169 hands, there are multiple ways to be dealt that hand. For example, an Eight Ten suited can be suited in spades (as shown above), or in diamonds, hearts, or clubs. Prior to the flop, all these combinations are equivalent.
- (45) I begin my analysis by setting aside one of the 169 possible starting hands that a player could be dealt. I then calculate every player's win rate (the amount won divided by the number of hands played) for all of the possible starting hands combined, *excluding the one I set aside*. That is, I

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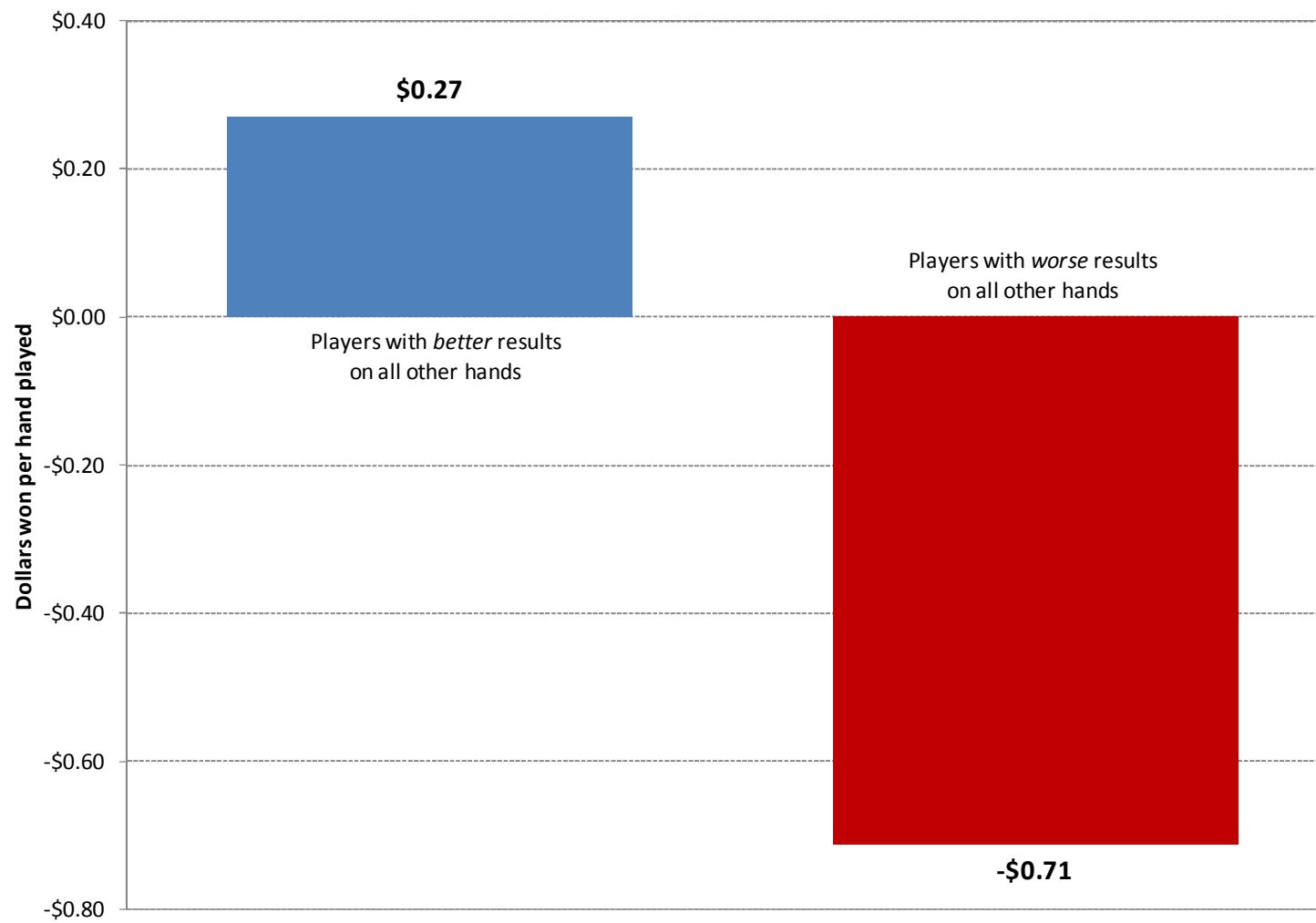
analyze each player's win rate on 168 of the 169 possible starting hands. I perform the analysis this way so that I can evaluate if players who are successful with one set of hands continue to be successful with a different set of hands.

- (46) For example, consider a starting hand of a Queen and a Jack suited ($Q\spadesuit J\spadesuit$).³ I start my analysis by setting that hand aside for all players in the data. I then evaluate how each player performed on all other starting hands in terms of their win rates (i.e., amounts won per hand played). I classify the players that finished in the top 50 percent of players in terms of their win rates as those with "better" outcomes. I classify the players in the bottom 50 percent in terms of their win rates as those with "worse" outcomes. Having established these two groups based upon all of the 168 possible starting hands other than $Q\spadesuit J\spadesuit$, I then compare the results of the two groups on those occasions that they were dealt $Q\spadesuit J\spadesuit$. If poker were a game of chance, I would not expect to see any correlation between winning with $Q\spadesuit J\spadesuit$ and winning with the other 168 hands. However, if players' skills, techniques, and strategies – i.e., how skillfully they play their hands – are driving their outcomes, then I would expect to see players who are relatively successful on all other hands to also be relatively successful with $Q\spadesuit J\spadesuit$.
- (47) In fact, I find that players who are identified as having better outcomes on all other hands continue to have better outcomes when considering $Q\spadesuit J\spadesuit$. Figure 4 below shows an example of the observed win rates for the two groups for $Q\spadesuit J\spadesuit$. As shown, at \$1/\$2 stakes, players with better results on all other hands win \$0.27 when dealt $Q\spadesuit J\spadesuit$; whereas players with less favorable results on all other hands lose \$0.71 when dealt $Q\spadesuit J\spadesuit$. This difference is statistically significant.

³ I denote a Queen and a Jack suited as $Q\spadesuit J\spadesuit$ in this section, but I am broadly referring to a Queen and a Jack suited of any suit, such as $Q\heartsuit J\heartsuit$.

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Figure 4: Win rate comparison: Queen Jack suited (e.g., Q ♠ J ♠)



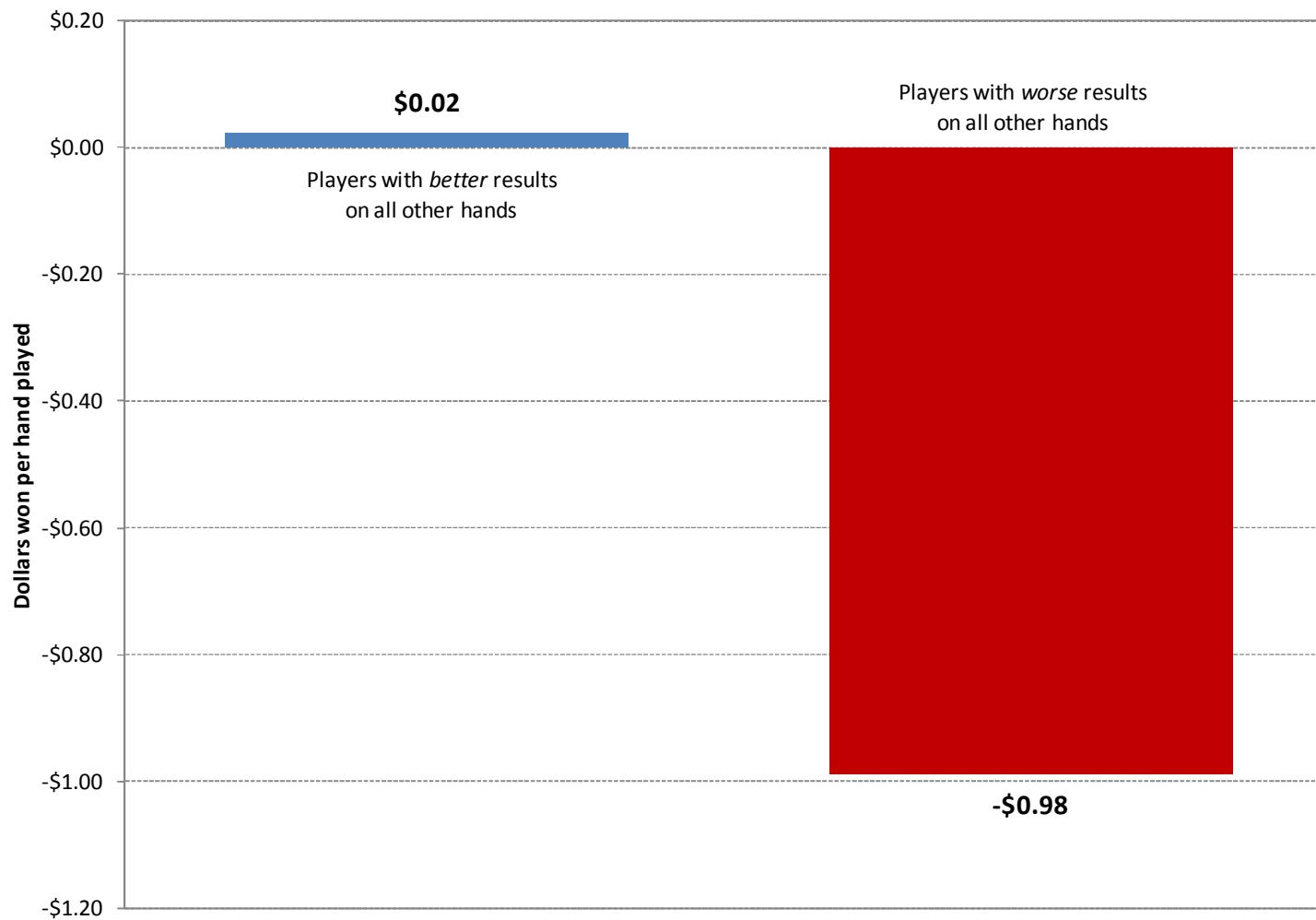
Note: Based on \$1/\$2 No-limit Hold'em; 6 max tables with 6 active players; all positions included; includes all Q J suited combinations.

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- (48) I perform this analysis for each of the 169 possible starting hands. Figure 5 through Figure 9 below show examples for five additional starting hands. All of these differences are statistically significant. As shown in the figures, in many cases, players with better results on all other hands have a positive win rate (i.e., they win money on average) with the hand of interest, whereas players with worse results on all other hands have a negative win rate (i.e., they lose money on average) with the hand of interest. In other cases, both players have positive win rates, but the players with better results on all other hands have a higher win rate than the players with worse results on all other hands. Finally, in other cases, both groups have negative win rates, but the players with better results on all other hands *lose less* on the hands of interest, suggesting that they are playing their hands more effectively. (Even very skillful players will lose money with many hands in the blind positions.)

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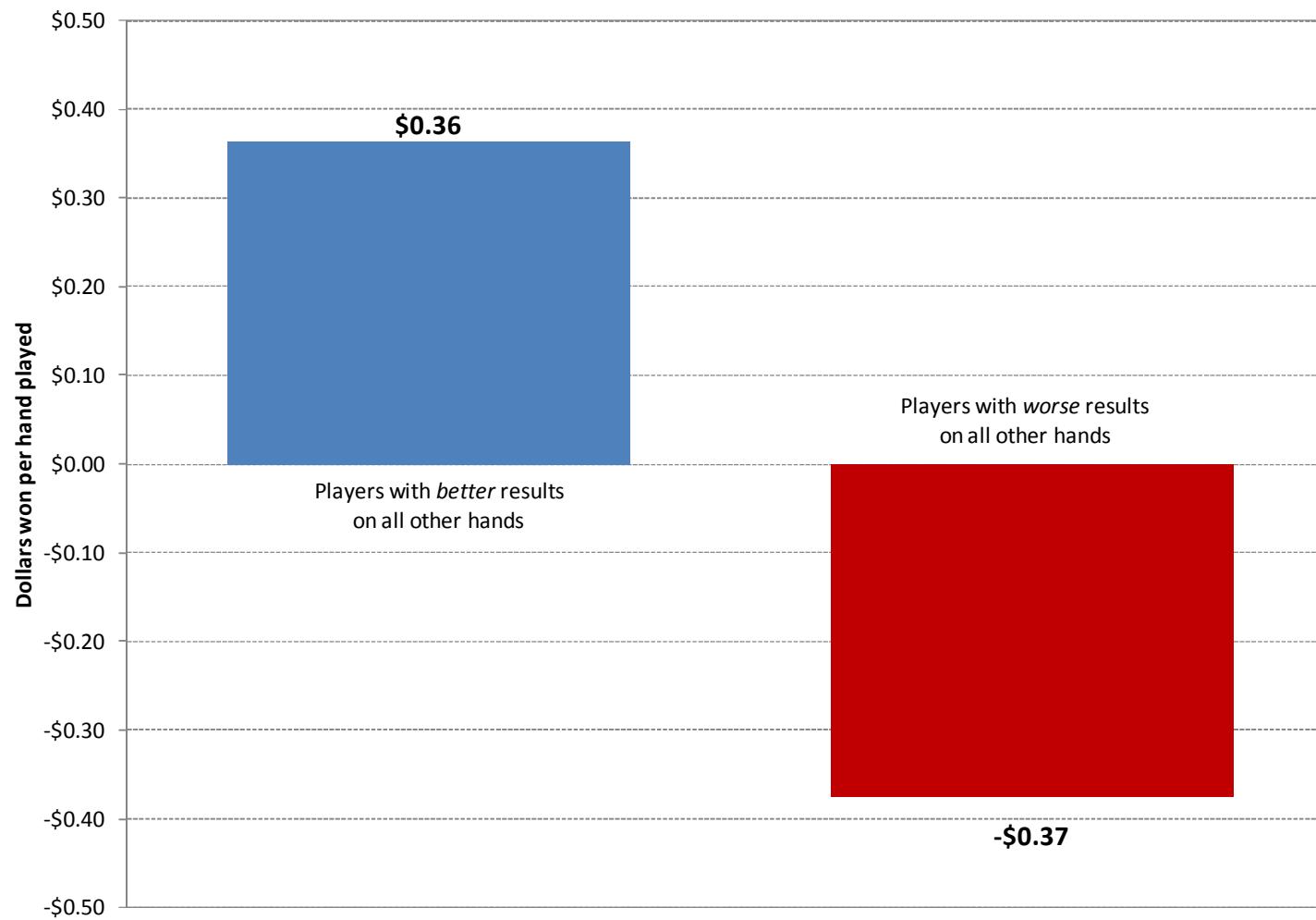
Figure 5: Win rate comparison: Ace Ten offsuit (e.g., A ♦ 10 ♣)



Note: Based on \$1/\$2 No-limit Hold'em; 6 max tables with 6 active players; all positions included; includes all A 10 offsuit combinations.

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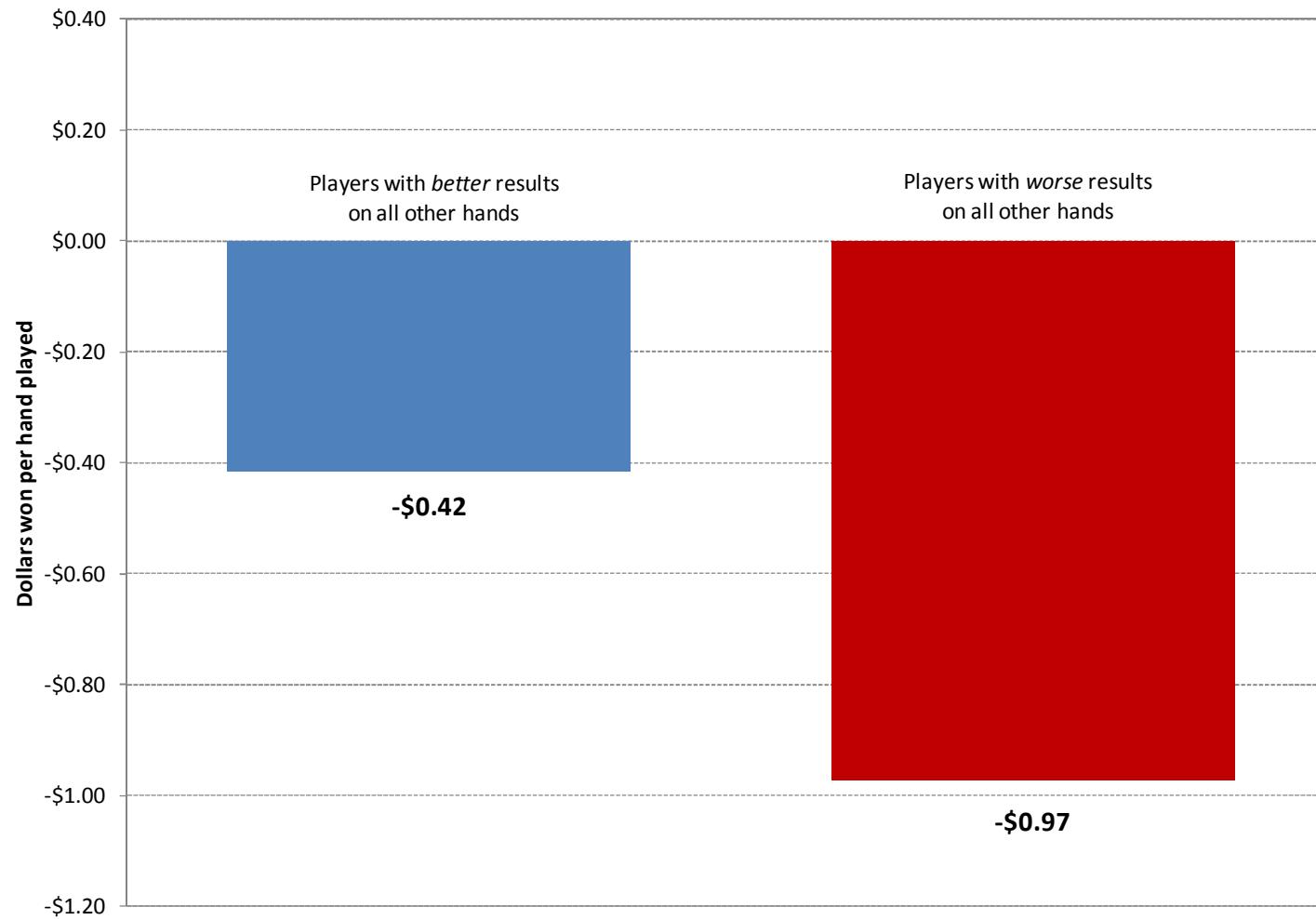
Figure 6: Win rate comparison: Pocket Sixes (e.g., 6♦ 6♣)



Note: Based on \$1/\$2 No-limit Hold'em; 6 max tables with 6 active players; all positions included; includes all pocket Sixes combinations.

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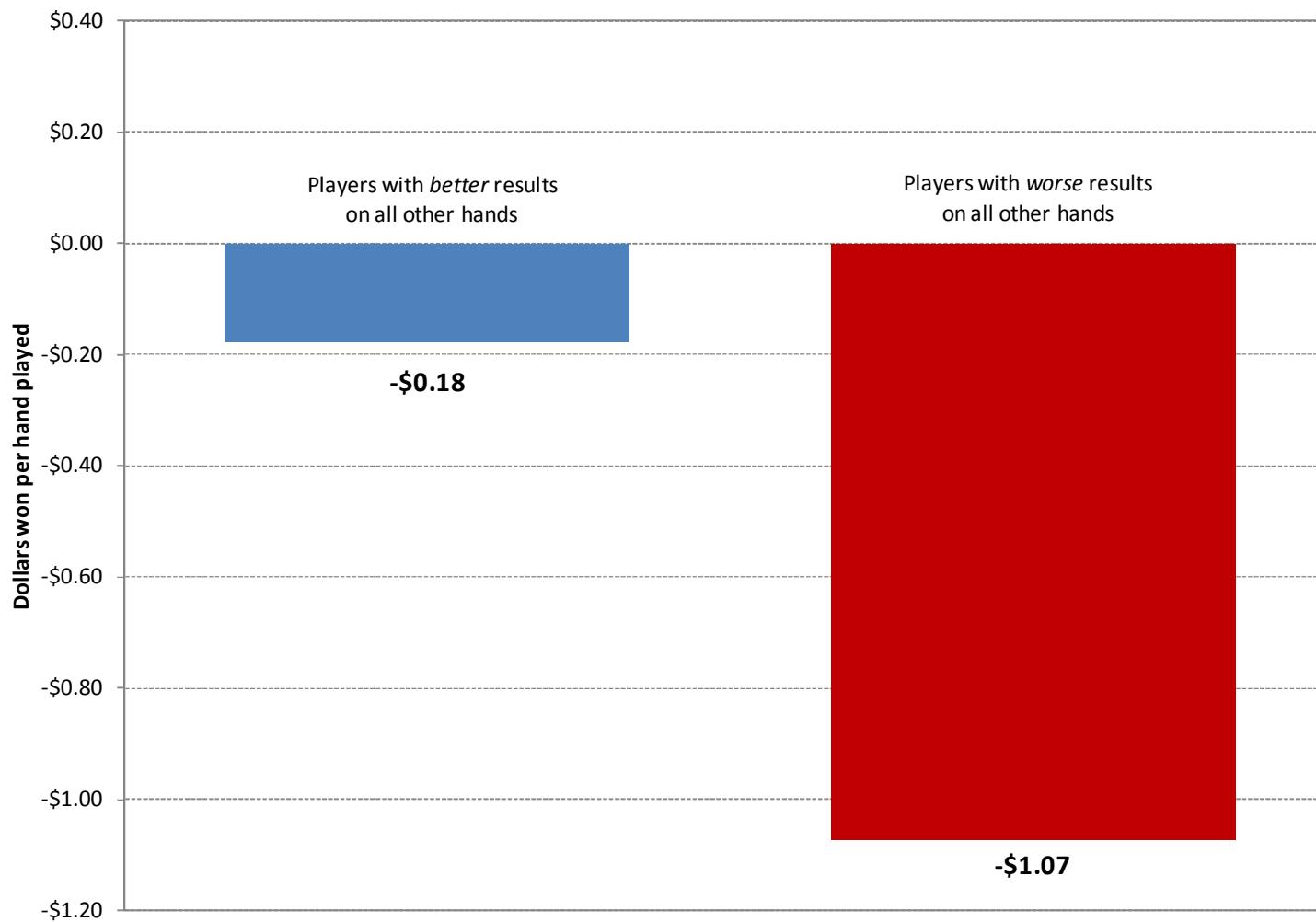
Figure 7: Win rate comparison: King Nine offsuit (e.g., K ♣ 9 ♦)



Note: Based on \$1/\$2 No-limit Hold'em; 6 max tables with 6 active players; all positions included; includes all offsuit K 9 combinations.

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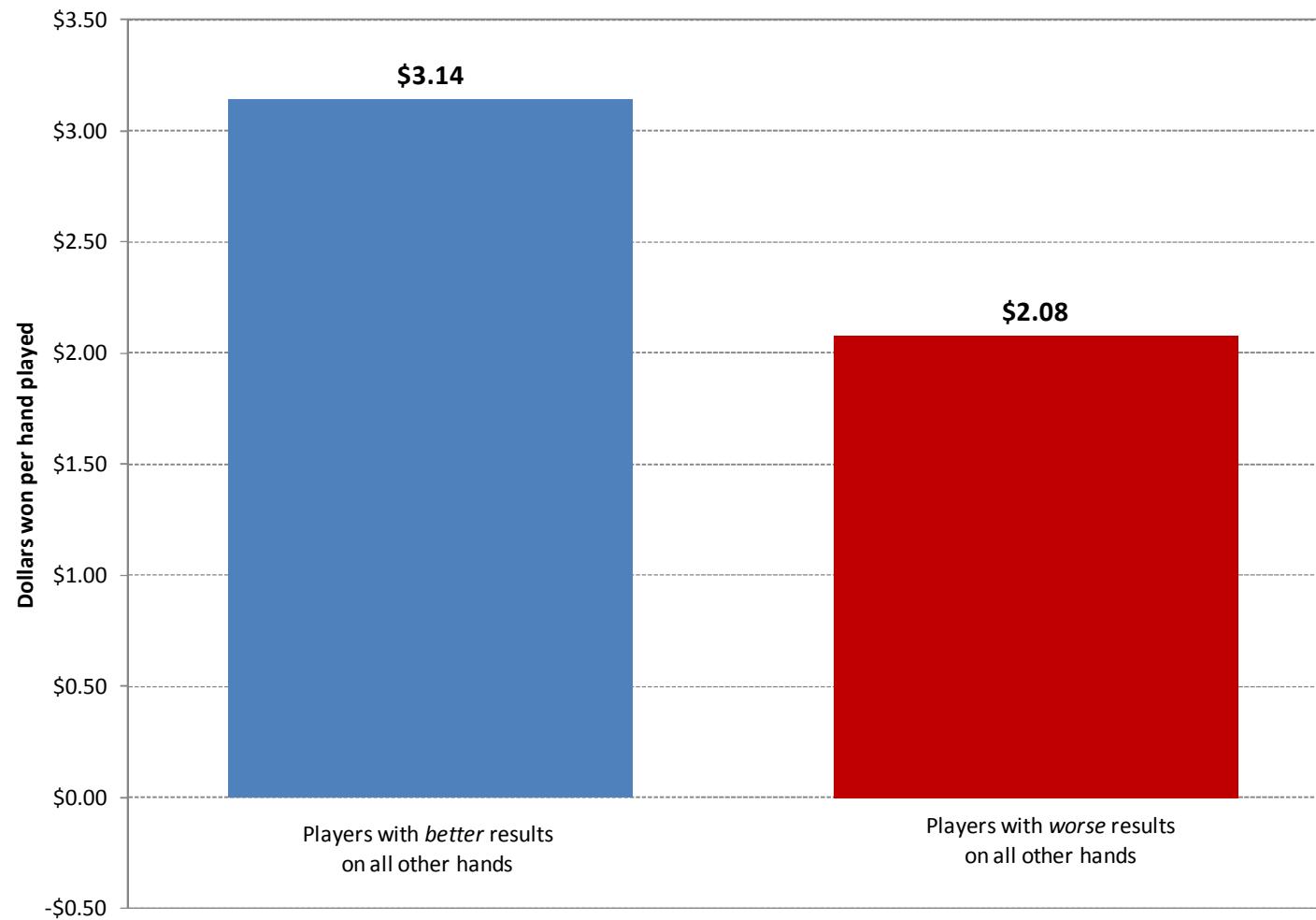
Figure 8: Win rate comparison: King Jack offsuit (e.g., K♦ J♣)



Note: Based on \$1/\$2 No-limit Hold'em; 6 max tables with 6 active players; all positions included; includes all offsuit K J combinations.

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Figure 9: Win rate comparison: Ace King offsuit (e.g., A ♣ K ♦)



Note: Based on \$1/\$2 No-limit Hold'em; 6 max tables with 6 active players; all positions included; includes all offsuit A K combinations.

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- (49) As shown in Figure 10 below, for players playing \$1/\$2 No Limit Hold 'em, I find that players identified as having better results on all other hands have a higher win rate for the particular hand of interest in *169 out of 169 possible starting hands*, and in 169 out of 169 cases, the difference between the two win rates is statistically significant. That is, in every hand to hand comparison, players with better win rates on the rest of the hands also have better win rates on the hand of interest. Again, if chance were the driving factor in poker outcomes, this relationship would not exist. Results would be random and uncorrelated across hands. In contrast, these results provide strong evidence that players' skill drives poker outcomes. This analysis provides independent corroboration for my opinion that skill predominates over chance in determining outcomes in poker. I find similar results for all of the stakes, as shown in the Appendix, Section IV.B.

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Figure 10: Win rates for players with better and worse results on all other hands for each of 169 possible starting hands, for \$1/\$2 stakes

Starting hand	Win rates for players with better results on all other hands		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
A A	\$18.64	\$16.52	\$0.15	✓
2 2	-\$0.25	-\$0.88	\$0.06	✓
3 3	-\$0.14	-\$0.87	\$0.06	✓
4 4	-\$0.03	-\$0.66	\$0.06	✓
5 5	\$0.15	-\$0.51	\$0.07	✓
6 6	\$0.36	-\$0.37	\$0.07	✓
7 7	\$0.61	-\$0.04	\$0.08	✓
8 8	\$1.09	\$0.30	\$0.08	✓
9 9	\$1.75	\$0.97	\$0.09	✓
T T	\$2.87	\$2.30	\$0.11	✓
J J	\$4.66	\$3.61	\$0.12	✓
Q Q	\$7.46	\$6.47	\$0.14	✓
K K	\$12.25	\$10.62	\$0.14	✓
K Q s	\$0.93	\$0.21	\$0.10	✓
K J s	\$0.48	\$0.08	\$0.09	✓
K T s	\$0.27	-\$0.54	\$0.08	✓
K 9 s	-\$0.16	-\$0.78	\$0.06	✓
K 8 s	-\$0.32	-\$0.77	\$0.05	✓
K 7 s	-\$0.34	-\$0.75	\$0.04	✓
K 6 s	-\$0.37	-\$0.76	\$0.04	✓
K 5 s	-\$0.41	-\$0.77	\$0.04	✓
K 4 s	-\$0.38	-\$0.89	\$0.03	✓
K 3 s	-\$0.40	-\$0.85	\$0.03	✓
K 2 s	-\$0.41	-\$0.92	\$0.03	✓
K A s	\$4.42	\$3.68	\$0.14	✓
Q J s	\$0.27	-\$0.71	\$0.09	✓
Q T s	\$0.07	-\$0.84	\$0.08	✓
Q 9 s	-\$0.21	-\$0.90	\$0.06	✓
Q 8 s	-\$0.36	-\$0.99	\$0.04	✓
Q 7 s	-\$0.43	-\$0.97	\$0.03	✓

Starting hand	Win rates for players with better results on all other hands		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
Q 6 s	-\$0.42	-\$0.83	\$0.03	✓
Q 5 s	-\$0.44	-\$0.91	\$0.03	✓
Q 4 s	-\$0.43	-\$0.80	\$0.03	✓
Q 3 s	-\$0.43	-\$0.83	\$0.03	✓
Q 2 s	-\$0.44	-\$0.76	\$0.02	✓
Q A s	\$2.25	\$1.43	\$0.12	✓
J T s	\$0.12	-\$0.49	\$0.09	✓
J 9 s	-\$0.17	-\$0.93	\$0.07	✓
J 8 s	-\$0.39	-\$1.02	\$0.05	✓
J 7 s	-\$0.44	-\$0.88	\$0.04	✓
J 6 s	-\$0.43	-\$0.88	\$0.03	✓
J 5 s	-\$0.43	-\$0.73	\$0.02	✓
J 4 s	-\$0.44	-\$0.86	\$0.02	✓
J 3 s	-\$0.45	-\$0.83	\$0.02	✓
J 2 s	-\$0.44	-\$0.81	\$0.02	✓
J A s	\$1.21	\$0.66	\$0.10	✓
T 9 s	-\$0.17	-\$0.75	\$0.08	✓
T 8 s	-\$0.29	-\$1.08	\$0.06	✓
T 7 s	-\$0.42	-\$0.92	\$0.05	✓
T 6 s	-\$0.42	-\$0.88	\$0.03	✓
T 5 s	-\$0.44	-\$0.83	\$0.02	✓
T 4 s	-\$0.45	-\$0.76	\$0.02	✓
T 3 s	-\$0.44	-\$0.73	\$0.02	✓
T 2 s	-\$0.44	-\$0.72	\$0.02	✓
T A s	\$0.75	\$0.04	\$0.09	✓
9 8 s	-\$0.36	-\$1.16	\$0.08	✓
9 7 s	-\$0.44	-\$0.86	\$0.06	✓
9 6 s	-\$0.44	-\$0.89	\$0.04	✓
9 5 s	-\$0.45	-\$0.83	\$0.03	✓
9 4 s	-\$0.45	-\$0.70	\$0.02	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
9 3 s	-\$0.45	-\$0.74	\$0.02	✓
9 2 s	-\$0.45	-\$0.73	\$0.02	✓
9 A s	\$0.21	-\$0.51	\$0.07	✓
8 7 s	-\$0.34	-\$0.95	\$0.07	✓
8 6 s	-\$0.42	-\$0.86	\$0.06	✓
8 5 s	-\$0.44	-\$0.92	\$0.04	✓
8 4 s	-\$0.48	-\$0.77	\$0.02	✓
8 3 s	-\$0.45	-\$0.75	\$0.02	✓
8 2 s	-\$0.45	-\$0.64	\$0.02	✓
8 A s	\$0.08	-\$0.65	\$0.07	✓
7 6 s	-\$0.37	-\$0.94	\$0.07	✓
7 5 s	-\$0.47	-\$0.91	\$0.05	✓
7 4 s	-\$0.45	-\$0.89	\$0.03	✓
7 3 s	-\$0.47	-\$0.74	\$0.02	✓
7 2 s	-\$0.46	-\$0.69	\$0.02	✓
7 A s	-\$0.06	-\$0.96	\$0.06	✓
6 5 s	-\$0.39	-\$1.00	\$0.06	✓
6 4 s	-\$0.47	-\$1.01	\$0.04	✓
6 3 s	-\$0.47	-\$0.85	\$0.03	✓
6 2 s	-\$0.47	-\$0.72	\$0.02	✓
6 A s	-\$0.16	-\$0.88	\$0.06	✓
5 4 s	-\$0.45	-\$1.01	\$0.06	✓
5 3 s	-\$0.47	-\$0.77	\$0.04	✓
5 2 s	-\$0.48	-\$0.76	\$0.03	✓
5 A s	-\$0.08	-\$0.82	\$0.07	✓
4 3 s	-\$0.49	-\$1.08	\$0.04	✓
4 2 s	-\$0.47	-\$0.87	\$0.03	✓
4 A s	-\$0.15	-\$0.83	\$0.06	✓
3 2 s	-\$0.50	-\$1.06	\$0.03	✓
3 A s	-\$0.18	-\$0.92	\$0.06	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
2 A s	-\$0.28	-\$0.94	\$0.06	✓
K Q o	\$0.10	-\$0.72	\$0.04	✓
K J o	-\$0.18	-\$1.07	\$0.03	✓
K T o	-\$0.30	-\$1.17	\$0.03	✓
K 9 o	-\$0.42	-\$0.97	\$0.02	✓
K 8 o	-\$0.42	-\$0.85	\$0.01	✓
K 7 o	-\$0.42	-\$0.73	\$0.01	✓
K 6 o	-\$0.43	-\$0.69	\$0.01	✓
K 5 o	-\$0.43	-\$0.60	\$0.01	✓
K 4 o	-\$0.43	-\$0.62	\$0.01	✓
K 3 o	-\$0.43	-\$0.63	\$0.01	✓
K 2 o	-\$0.43	-\$0.58	\$0.01	✓
K A o	\$3.14	\$2.08	\$0.08	✓
Q J o	-\$0.31	-\$1.22	\$0.03	✓
Q T o	-\$0.36	-\$1.21	\$0.02	✓
Q 9 o	-\$0.42	-\$0.90	\$0.02	✓
Q 8 o	-\$0.43	-\$0.76	\$0.01	✓
Q 7 o	-\$0.43	-\$0.68	\$0.01	✓
Q 6 o	-\$0.43	-\$0.64	\$0.01	✓
Q 5 o	-\$0.43	-\$0.59	\$0.01	✓
Q 4 o	-\$0.43	-\$0.55	\$0.01	✓
Q 3 o	-\$0.43	-\$0.56	\$0.01	✓
Q 2 o	-\$0.44	-\$0.56	\$0.00	✓
Q A o	\$1.11	\$0.32	\$0.06	✓
J T o	-\$0.39	-\$1.23	\$0.03	✓
J 9 o	-\$0.43	-\$0.95	\$0.02	✓
J 8 o	-\$0.44	-\$0.76	\$0.01	✓
J 7 o	-\$0.43	-\$0.64	\$0.01	✓
J 6 o	-\$0.44	-\$0.56	\$0.01	✓
J 5 o	-\$0.44	-\$0.58	\$0.01	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
J 4 o	-\$0.44	-\$0.54	\$0.00	✓
J 3 o	-\$0.44	-\$0.54	\$0.00	✓
J 2 o	-\$0.44	-\$0.53	\$0.00	✓
J A o	\$0.35	-\$0.57	\$0.04	✓
T 9 o	-\$0.43	-\$1.06	\$0.02	✓
T 8 o	-\$0.44	-\$0.76	\$0.01	✓
T 7 o	-\$0.44	-\$0.64	\$0.01	✓
T 6 o	-\$0.44	-\$0.56	\$0.01	✓
T 5 o	-\$0.44	-\$0.56	\$0.00	✓
T 4 o	-\$0.44	-\$0.54	\$0.00	✓
T 3 o	-\$0.44	-\$0.50	\$0.00	✓
T 2 o	-\$0.44	-\$0.53	\$0.00	✓
T A o	\$0.02	-\$0.98	\$0.04	✓
9 8 o	-\$0.46	-\$0.97	\$0.02	✓
9 7 o	-\$0.44	-\$0.73	\$0.01	✓
9 6 o	-\$0.44	-\$0.62	\$0.01	✓
9 5 o	-\$0.44	-\$0.56	\$0.01	✓
9 4 o	-\$0.44	-\$0.52	\$0.00	✓
9 3 o	-\$0.44	-\$0.50	\$0.00	✓
9 2 o	-\$0.44	-\$0.50	\$0.00	✓
9 A o	-\$0.27	-\$1.04	\$0.02	✓
8 7 o	-\$0.46	-\$0.89	\$0.01	✓
8 6 o	-\$0.44	-\$0.68	\$0.01	✓
8 5 o	-\$0.44	-\$0.62	\$0.01	✓
8 4 o	-\$0.44	-\$0.54	\$0.00	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
8 3 o	-\$0.44	-\$0.53	\$0.00	✓
8 2 o	-\$0.44	-\$0.51	\$0.00	✓
8 A o	-\$0.33	-\$1.14	\$0.02	✓
7 6 o	-\$0.46	-\$0.79	\$0.01	✓
7 5 o	-\$0.44	-\$0.67	\$0.01	✓
7 4 o	-\$0.44	-\$0.57	\$0.01	✓
7 3 o	-\$0.44	-\$0.51	\$0.00	✓
7 2 o	-\$0.45	-\$0.53	\$0.00	✓
7 A o	-\$0.40	-\$1.16	\$0.02	✓
6 5 o	-\$0.45	-\$0.80	\$0.01	✓
6 4 o	-\$0.44	-\$0.60	\$0.01	✓
6 3 o	-\$0.45	-\$0.55	\$0.01	✓
6 2 o	-\$0.45	-\$0.51	\$0.00	✓
6 A o	-\$0.43	-\$1.15	\$0.02	✓
5 4 o	-\$0.46	-\$0.67	\$0.01	✓
5 3 o	-\$0.45	-\$0.58	\$0.01	✓
5 2 o	-\$0.45	-\$0.56	\$0.01	✓
5 A o	-\$0.40	-\$0.96	\$0.02	✓
4 3 o	-\$0.46	-\$0.67	\$0.01	✓
4 2 o	-\$0.45	-\$0.54	\$0.01	✓
4 A o	-\$0.41	-\$0.99	\$0.01	✓
3 2 o	-\$0.45	-\$0.59	\$0.01	✓
3 A o	-\$0.42	-\$0.98	\$0.01	✓
2 A o	-\$0.44	-\$0.97	\$0.01	✓

of 169 starting hands in which players with *better* results on all other hands have statistically significant higher win rates: 169# of 169 starting hands in which players with *worse* results on all other hands have statistically significant higher win rates: 0

Note: Based on \$1/\$2 No-limit Hold'em; 6 max tables with 6 active players; all positions included.

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- (50) Note that the analyses shown above are for 6-handed games played at 6-player max tables. I performed my analysis this way because strategies can vary for a particular player when playing 6-handed versus short-handed. Thus, limiting to 6-handed games played at 6-player max tables provides a common comparison of players in similar situations. I also performed my analysis separately for different stakes (i.e., \$10/\$20 games were considered separately from \$1/\$2 games). A player's relative skill may vary across stakes – for example, a dominant player in a \$1/\$2 game may be an average player at a higher stakes game since the players at higher stakes tend to be more skillful.
- (51) I have implemented the analysis described above for each of the stakes provided in the PokerStars data and find results that are consistent with those described above. Figure 11 below shows the results of this analysis for each of the stakes. These results confirm my finding that skill predominates over chance in poker.

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Figure 11: Comparison of win rates for all starting hands for all stakes

Stakes	Number of starting hands with which players with <i>better</i> results on all other hands have statistically significant higher win rates	Number of starting hands with which players with <i>worse</i> results on all other hands have statistically significant higher win rates
\$0.50/ \$1.00	169	0
\$1/\$2	169	0
\$2/\$4	167	0
\$3/\$6	161	0
\$5/\$10	144	0
\$10/\$20	51	1

Note: 6 max tables with 6 active players; all positions included.

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III.E. Statistical analysis of skill in poker

III.E.1. Measuring poker skill – construction of skill index

- (53) In this section, I discuss a statistical test I have implemented to determine whether or not skill predominates over chance in poker. In order to conduct the statistical test, I first constructed what I refer to as a “skill index” for each of the players in the data. I construct this skill index for each player independently for each different stakes level, because the strategies that comprise skillful play vary depending upon the stakes. The average opponent is more skillful and more experienced the higher in stakes a player advances, and the strategies required to beat experienced and skillful players differ from those which are optimal against less skillful players.
- (54) To assign a skill index to each player, I first divided the players into two groups by randomly assigning half the players to an “evaluation group” and the other half to a “prediction group.” The evaluation group is used to construct a formula for an index of skill by identifying skill and strategy attributes that are associated with success. I included in this analysis 241 different statistics about each player, capturing characteristics pertaining to the strategies that players tend to employ. These statistics are described in detail below in Section III.E.2.
- (55) To construct the skill index from these statistics, I use a common statistical procedure known as multiple regression analysis.⁴ Multiple regression analysis is a standard technique used in a wide variety of settings to determine how one set of factors (in this case, statistics that measure characteristics of a player’s strategy and skill) affects another variable (in this case, poker success as measured by the player’s per-hand win rate). This is a common statistical technique for evaluating the relationship between participants’ skills and outcomes in a wide variety of fields of endeavor.⁵
- (56) I use regression analysis to determine the formula to apply to the 241 statistics that does the best job of matching predicted skill to observed win rates among the players in the evaluation group. For example, one of the statistics measures the player’s “aggression factor” on the third betting round, which is the number of times a player bets or raises divided by the number of times she calls, on that betting round. The regression analysis determines the value for this statistic that best predicts a high

⁴ James H. Stock and Mark W. Watson, *Introduction to Econometrics* (2nd Edition 2007, Addison-Wesley), p. 7. (“The conceptual framework used in this book is the multiple regression model, the mainstay of econometrics. This model...provides a mathematical way to quantify how a change in one variable affects another variable, holding other things constant.”)

⁵ For example, see: “Reading and spelling skills in the first school years predicted from phonemic awareness skills in kindergarten”, Lundberg, I., Olofsson, A., and Wall, S., *Scandinavian Journal of Psychology*, Volume 21, Issue 1, pages 159–173, September 1980; Tarter, B., et al, “Use of the Sports Performance Index for Hockey (SPI-H) to Predict NHL Player Value,” *International Journal of Performance Analysis in Sport*, Volume 9, Number 2, August 2009 , pp. 238-244; Hakes, J. K., Sauer, R., D., “An Economic Evaluation of the Moneyball Hypothesis,” *The Journal of Economic Perspectives*, Volume 20, Number 3, Summer 2006 , pp. 173-185.

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win rate. It also determines how much of a “penalty” to the skill index should be assigned to players who deviate from this value.⁶ Players who play aggressively either more or less often than this are judged to be less skillful than players who play aggressively the ideal fraction of the time. All of the skill statistics described above are assigned a relative weighting in the skill index, and the resulting formula can be applied to any poker player for whom those statistics are available.

- (57) Once I have a formula for the skill index based upon the actual results from the evaluation group, I then use that same formula to construct the skill index for the players in the prediction group. In this way, the formula for skill used in the prediction group does not depend in any way upon the actual winning or losing results of the players in the prediction group, since the formula was determined with reference only to the evaluation group.
- (58) Once I have applied the formula for the skill index to the players in the prediction group, I rank each of the players in the prediction group from highest to lowest in terms of how they scored on their skill index. I then divide the players in the prediction group into higher skilled and lower skilled groups depending on where each player lies on the skill index continuum. Specifically, I characterize players in the top 50% of the skill index as higher skilled and players in the bottom 50% as lower skilled.

III.E.2. Statistics used to measure skill

- (59) A number of different statistics were constructed to measure various aspects of a player’s strategic decision making throughout the course of play. Each of these basic statistics were then calculated for different circumstances, such as an initial hole card holding or position relative to the blinds, reflecting the fact that optimal strategy will vary depending upon these circumstances. The following six statistics are based on pre-flop decisions:
 - Voluntarily put money in pot percentage (VPIP): This measures the percentage of total hands played in which a player voluntarily (i.e., not by virtue of having posted the small or big blind) put money into the pot prior to seeing the flop. This provides a measure of how selective a player is in choosing which of her starting hands to play and which to fold. When used in conjunction with information about a player’s position relative to the blinds and initial card holding, this variable also reflects the degree to which a player is adjusting her strategy to reflect subtle differences in her situation.
 - Pre-flop raise percentage (PFR): This variable measures how aggressively or passively the player plays. It is calculated by first determining the number of hands in which the player raised pre-flop and then dividing this by the number of hands the player was dealt.

⁶ The details of the regression analysis are available upon request.

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- Pre-flop aggression ratio (PFAR): This variable also measures how aggressively or passively the player plays. It is calculated by dividing the PFR by the VPIP. The result is the percentage of the time the player led the pre-flop betting action when voluntarily playing a hand.
- Pre-flop three-bet percentage: A pre-flop three-bet occurs when a player makes an additional raise on top of another player's initial raise. The term three-bet originates from the notion that the forced blind bet is the first bet, the initial raise is the second bet, and the player's additional raise is the third bet. This statistic is calculated by dividing the number of times a player made a three-bet by the total number of hands that player was dealt.
- Pre-flop four-bet percentage: A raise of a three-bet is known as a four-bet. This statistic is calculated by dividing the number of times a player four-bet by the total number of hands that player was dealt.
- Pre-flop open limp percentage: An "open limp" is the act of calling, as opposed to raising, the big blind when a player elects to play in an unraised pot. The open limp percentage measures the frequency of this strategy, and often also reflects upon a player's general aggressiveness and starting hand selection. This statistic is calculated by dividing the number of hands in which a player open limps by the total number of hands that player was dealt.

(60) The following four statistics measure aspects of a player's strategic decisions during the three remaining betting rounds: the flop, the turn, and the river:

- Flop aggression percentage: This statistic measures the ratio of aggressive flop decisions to passive flop decisions, and is calculated by dividing the number of flop bets and flop raises by flop calls.
- Turn aggression percentage: This statistic measures the ratio of aggressive turn decisions to passive turn decisions, and is calculated by dividing the number of turn bets and turn raises by turn calls.
- River aggression percentage: This statistic measures the ratio of aggressive river decisions to passive river decisions, and is calculated by dividing the number of river bets and river raises by river calls.
- Went to showdown percentage: This statistic is the ratio of two intermediate statistics, Flops Seen (FS) and Showdowns Seen (SS). FS is a count of the number of hands in which the player was still active in the hand (i.e., had not folded) until at least the flop was dealt. SS is a count of hands in which the player was still active in the hand (i.e., had not folded) when the hand was completed by a showdown on the river. The ratio of SS/FS, defines the percentage of times a player goes to showdown when she sees a flop.

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- (61) For all ten of the preceding statistics, reflecting both pre-flop and subsequent decisions, a number of variations are included in the regression and resulting skill index. For each of the ten statistics, both the statistic itself and the statistic multiplied by itself are included. This is a standard approach in regression analysis, though the explanation for including these “squared terms” is somewhat technical. Essentially, the inclusion of the squared terms causes the regression analysis to find the ideal level of each statistic, and to penalize a player more as her play deviates more from that ideal level. Thus there are twenty underlying statistics that enter into subsequent analyses.
- (62) For each of the twenty resulting statistics (the original ten plus the squared term for each), a separate version of each statistic is calculated for each of the six positions, relative to the big blind. Thus, for example, the measures of a player’s selectivity and aggressiveness in the big blind are different from those of the same player in the small blind, last position, or any other position.
- (63) In addition, for each of the twenty underlying statistics a separate version of each statistic is calculated to reflect the player’s strategy with each of five groups of starting hands. The five groups of starting hands are shown in Figure 12.

Figure 12: Hand groups for all 169 possible starting hands

Hand group	Starting hand																	
Group I	AA 22 33 44 55 66 77 88 99 TT JJ QQ KK KQs AKs AQs AJs ATs KQo AKo AQo AJo ATo																	
Group II	KJs KTs QJs QTs JTs T9s KJo KTo QJo QTo JTo T9o																	
Group III	9As 8As 7As 6As 5As 4As 3As 2As 9Ao 8Ao 7Ao 6Ao 5Ao 4Ao 3Ao 2Ao																	
Group IV	K9o K8o K7o K6o K5o K4o K3o K2o Q9o Q8o Q7o Q6o Q5o Q4o Q3o Q2o J9o J8o J7o J6o J5o J3o J2o T8o T7o T6o T5o T4o T3o T2o 98o 97o 96o 95o 94o 93o 92o 87o 86o 85o 84o 83o 82o 76o 75o 74o 73o 72o 65o 64o 63o 62o 54o 53o 52o 43o 42o 32o																	
Group V	K9s K8s K7s K6s K5s K4s K3s K2s Q9s Q8s Q7s Q6s Q5s Q4s Q3s Q2s J9s J8s J7s J6s J5s J4s J3s J2s T8s T7s T6s T5s T4s T3s T2s 98s 97s 96s 95s 94s 93s 92s 87s 86s 85s 84s 83s 82s 76s 75s 74s 73s 72s 65s 64s 63s 62s 54s 53s 52s 43s 42s 32s J4o																	

- (64) Thus there are $20 \times (1 \text{ overall} + 6 \text{ position specific} + 5 \text{ hand-group specific}) = 240$ possible strategy statistics. Since it is impossible to open limp when one starts in the big blind position, two of these 240 statistics do not exist (the open limp percentage and its square). Finally, three additional sources of information about each player that might reflect upon their playing skill are constructed from the database.
 - Engage in chat flag: Players have the option to talk to one another through the chat window at each table. I flag players based on whether they have ever participated in chat at the table for purposes of analyzing whether chat is an indication of a lack of focus on the game going on

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around them, or whether it is indicative of a familiarity with the game and a comfort level at the table.

- Premature post flag: When a player joins a 6-max table, it is least costly to do so in the big blind position. Joining in any other position forces the player to post the big blind early rather than waiting until the blind comes around to the player in turn. Thus, if someone posts prematurely, it may be a sign of impatience, which often characterizes low skill. To account for the possibility of unintentional posts by clicking the wrong option on the computer screen, or a conscious decision to join the game early due to the possibility of a very weak player being involved in the hand, I count players who post prematurely less than 1 out of every 10,000 hands as having never posted prematurely.
- Auto-rebuy flag: When a player drops below the maximum buy-in allowed for a given table, they have the option to add additional chips to get back to the table maximum. This can either be done by submitting a request manually, or automatically, by changing a setting in the PokerStars preferences menu. Players who set their preferences to automatically rebuy are flagged as such in the data. There may be varying interpretations of this variable, but it clearly indicates a strategic choice made by the player.

- (65) In total, there are 241 strategy and decision statistics included in the calculation of each player's skill index at a particular stakes level.

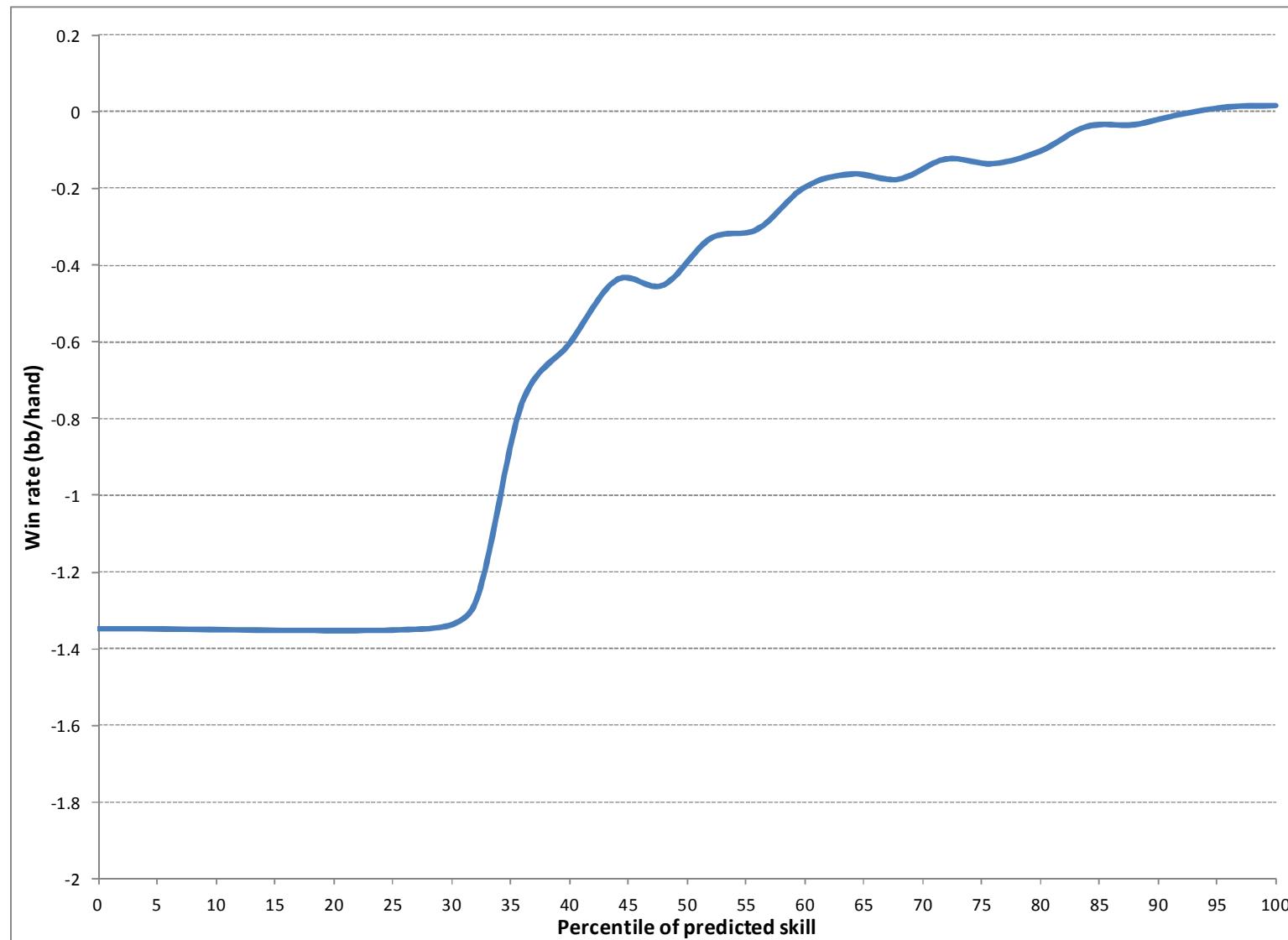
III.E.3. Evaluation of skill measurement

- (66) Having established a formula for a skill index and having applied it to the prediction group, the question naturally arises: how well does the skill index predict actual player performance? Figure 13 below shows the relationship between "predicted skill" and the actual win rate for all players in the prediction group of the \$5/\$10 stakes games. The line slopes upward to the right, indicating that players with higher predicted skill on average have higher win rates.⁷ If poker were a game of chance, there would be no relationship between predicted skill and results. The slope of the line reflects the much higher expected win rate of skillful players compared to less skillful players. This relationship characterizes games of skill. I find similar results for all of the stakes, as shown in the Appendix, Section IV.C.

⁷ Players with too few hands played to calculate statistics for all the various hand and position combinations are assigned the lowest skill level, accounting in large part for the flat portion of the curve on the left. Details of the skill index are available upon request.

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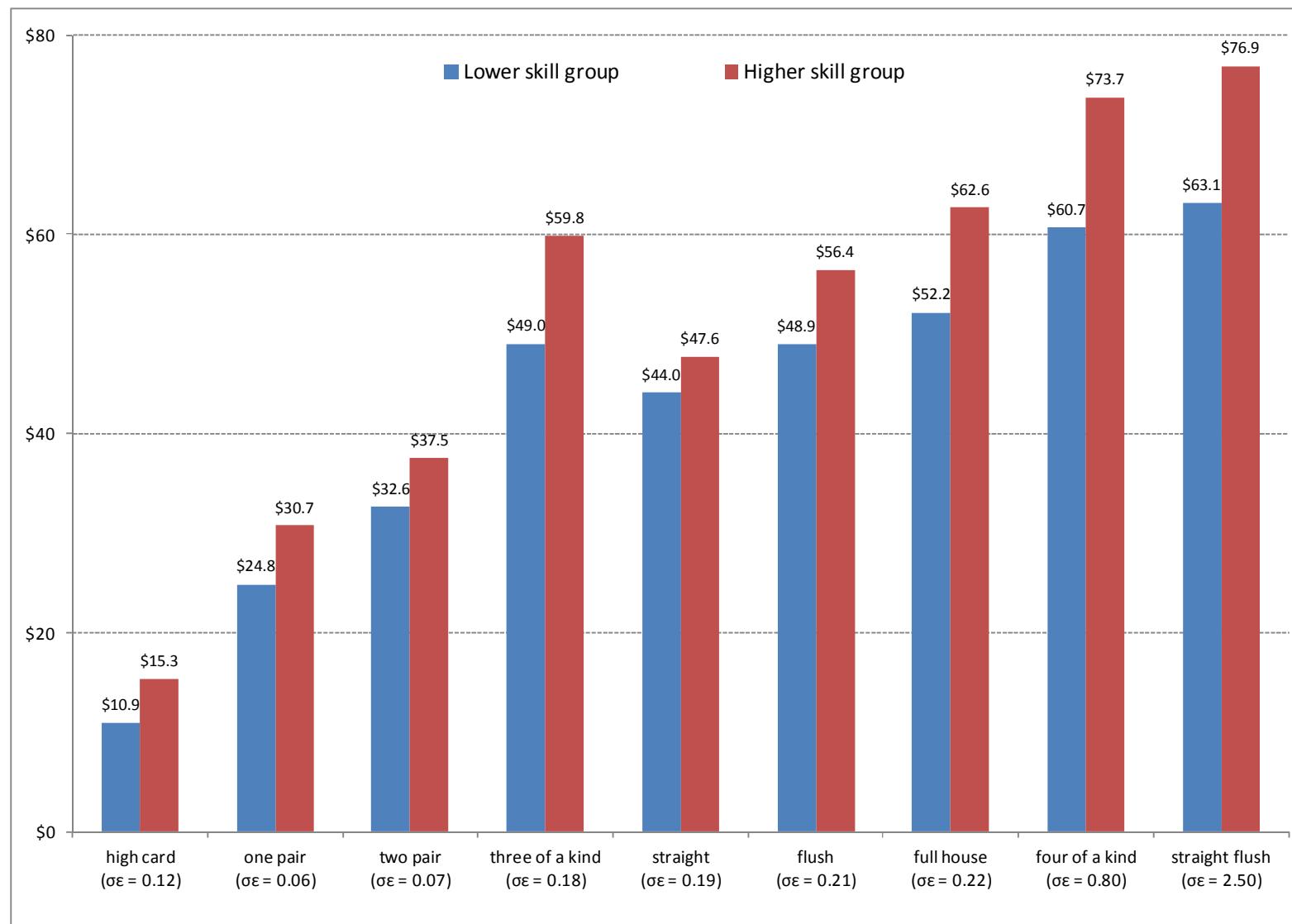
Figure 13: Average win rate for players of different predicted skill, for \$5/\$10 stakes players in the prediction group



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- (67) I have also assessed whether the players of higher skill have better outcomes than the players of lower skill when they make the exact same hand. That is, do players of higher skill win more when they show down a flush, for instance, than players of lower skill? In order to perform this analysis, I looked only at hands in which the winner of the hand won at showdown (meaning any and all bets were called and the winning hand exposed for the table to see). This set of hands was chosen because the PokerStars data identify the winning hand when the hand goes to showdown. This allowed me to categorize each winning hand at showdown into one of nine categories. Losing hands and hands that did not reach showdown are excluded from this analysis.
- (68) This analysis addresses the question of whether higher skilled players win more because they squeeze additional value out of every situation. Figure 14 below shows that players of higher skill (i.e., top 50% of players on the skill index) have a higher win rate with every possible hand category at showdown than players of lower skill, at \$1/\$2 stakes. The differences between the higher skilled and lower skilled groups are statistically significant for all possible winning hands. This demonstrates that the more skillful players are playing their hands better – i.e., employing superior strategies and techniques – relative to the lower skilled players, resulting in better outcomes. I find similar results for all of the stakes, as shown in the Appendix, Section IV.D.

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Figure 14: Win rates for higher skilled and lower skilled players with the same hand category, \$1/\$2 stakes

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III.E.4. Determining if skill predominates over chance

- (69) I next determine whether skill predominates over chance in poker by testing whether more skillful players predominate over less skillful players. I approach this question by asking: “if a higher skilled player and a lower skilled player both played the same number of hands, how many hands would it take before the more skillful player is at least 90% likely to be ahead of (i.e., to have won more money or lost less money than) the less skillful player?”
- (70) To answer this question, I conduct what is essentially a controlled competition between the more skillful and less skillful players. To visualize this competition, imagine that a more skillful player and a less skillful player are playing at side by side tables, playing the same number of hands against other randomly selected players. The winner of the competition between the two players is the one who has won more (or lost less) than the other after some number of hands. In order to simulate this type of competition using the data I have available, I use a technique known as “Monte Carlo simulation.” Like regression analysis, Monte Carlo simulation is a standard tool for determining the outcomes of hypothesized interactions, and is widely used in published studies in economics and the physical sciences.⁸ It is a technique well suited to answer questions regarding a competition involving a series of poker hands.
- (71) I first choose the “teams” for my competition. To do so, I divide the prediction group into halves, with the top 50% of all players by predicted skill in the higher skilled group and the bottom 50% of all players by estimated skill in the lower skilled group. I then randomly select one hand out of all of the higher skilled players’ actual hands played and similarly one hand from all of the lower skilled players’ actual hands played.⁹ I then record the results. For example, in my first draw, I may find that the hand randomly selected from among the higher skilled players involves a hand in which a higher skilled player loses \$6. I may find that the hand randomly drawn from the lower skilled players involves a hand in which a lower skilled player loses \$4. If I stopped my simulation here, after only one hand, I would find that the lower skilled player is ahead of the higher skilled player since the higher skilled player has lost more than the lower skilled player. However, if I continue on with another hand, I may find that in the next hand randomly drawn the higher skilled player wins \$8 and the lower skilled player loses \$2. At this point, the higher skilled player’s cumulative result is positive \$2 (she lost \$6 on the first hand but won \$8 on the second) and the lower skilled player’s cumulative result is a loss of \$6 (she lost \$4 on the first hand and lost an additional \$2 on the second). As a result, in this hypothetical example, after one hand the lower skilled player was ahead, but after two hands the higher skilled player was ahead.

⁸ See for example, Judd, Kenneth L., Numerical methods in economics, MIT Press (1998) p.305. A Google Scholar search of the exact phrase “Monte Carlo simulation” in scholarly articles in Business, Administration, Finance and Economics yields over 39,000 hits on books and articles containing the phrase.

⁹ Because there are so many hands played by both groups, I first create a sample of up to 6 million hands for each skill category and then randomly draw from this sample of hands. Details of this analysis are available upon request.

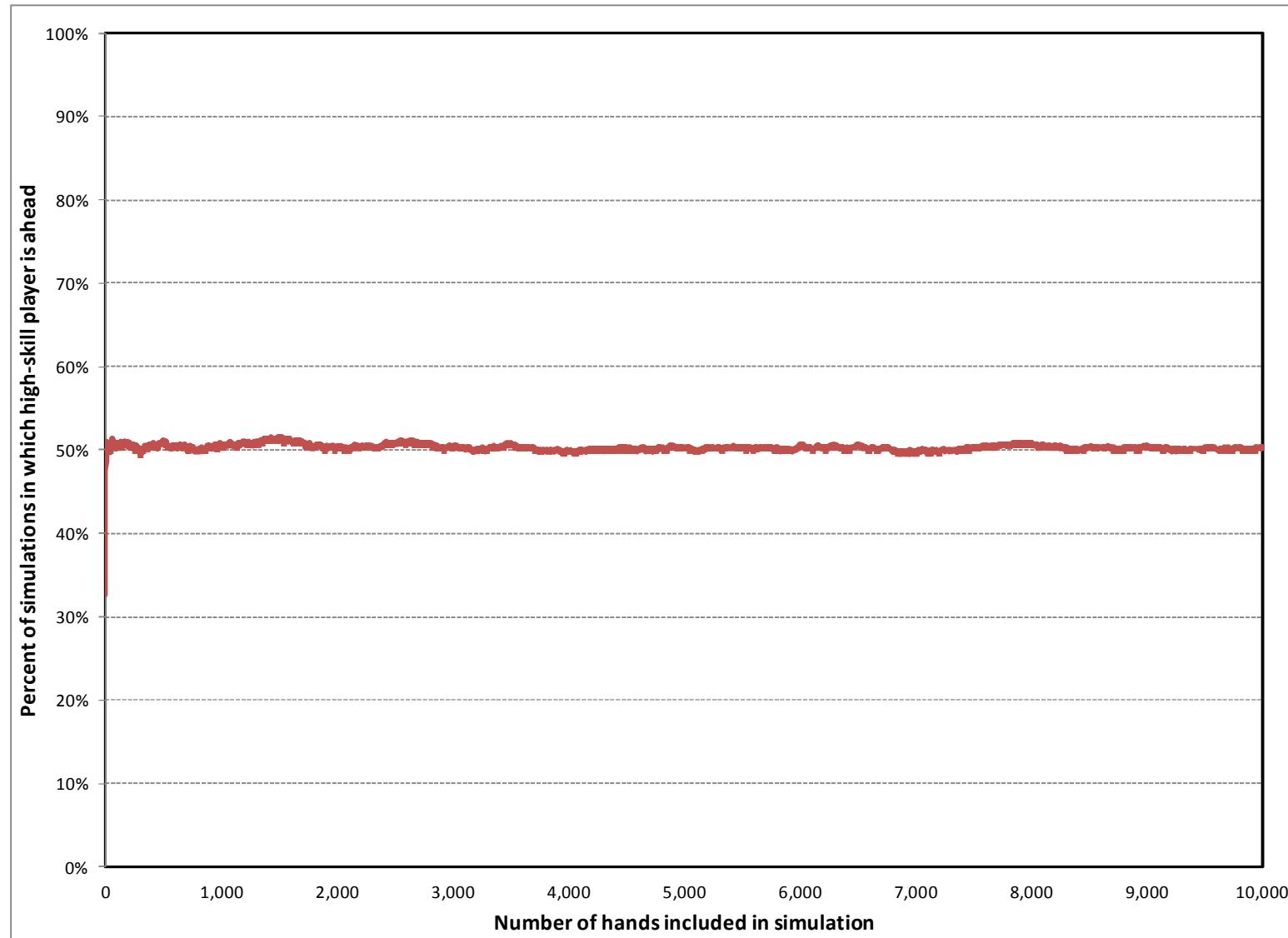
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- (72) To conduct my simulation, I repeat the process described above over and over again, cycling repeatedly through the big blind, small blind, and other positions until 10,000 hands have been randomly selected for both the higher and lower skill player groups. At each hand – one through 10,000 – I record which player is ahead on a cumulative basis, the lower skilled or higher skilled player. This constitutes one simulation. I then run 10,000 such simulations, and track the fraction of these in which the higher skill group is ahead of the lower skill group after each hand.
- (73) Before delving into the results, it is important to first consider what one would expect to find if poker is a game of chance versus what one would expect to find if poker is a game of skill. If poker were a game of chance, then my skill groupings would be meaningless since relative skill would be irrelevant for the outcomes. As a result, there would be no reason to expect the higher skill group or the lower skill group to dominate the other. Rather, it would be similar to flipping coins – the “higher skill group” may be ahead in some simulations based upon chance and behind in others based upon chance. We would thus expect the higher skilled player to be ahead of the lower skilled player in about 50% of the simulations if poker were a game of chance. Alternatively, if poker is a game of skill, we would expect major differences in outcomes between the higher skilled and lower skilled groups to develop. In particular, while the results may be unpredictable after only a few hands, we would expect to see the higher skilled group “pull away” from the lower skill group as more hands are played. Ultimately, after some number of hands, we would expect the higher skill group to be ahead in the vast majority of the simulations since the higher skilled players will be playing their hands more effectively – winning more money with strong hands and losing less with weak hands.
- (74) Figure 15 below shows what the results of my simulation would look like if poker were a game of chance. The number of simulations in which the higher skilled player is ahead would hover around 50% throughout all hands played since skill would be irrelevant, and thus the players identified as higher skilled would be no different than those identified as lower skilled. Of course, chance outcomes will result in slight deviations from the expected 50% outcome, but since skill would be a non-factor, neither group would be expected to dominate over the other. It is worth noting that to construct Figure 15, I performed the simulation described above for \$1/\$2 stakes, but randomly selected hands irrespective of player skill.¹⁰ That is, I set up my simulations to randomly select hands from the pool of all available hands, but did not distinguish higher skilled and lower skilled players. As a result, Figure 15 shows the results of a game of poker in which I have “assumed away” the skill component.

¹⁰ I performed this same simulation for all other stakes and, as expected, the results are virtually identical.

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Figure 15: Expected results of simulations if poker were a game of chance

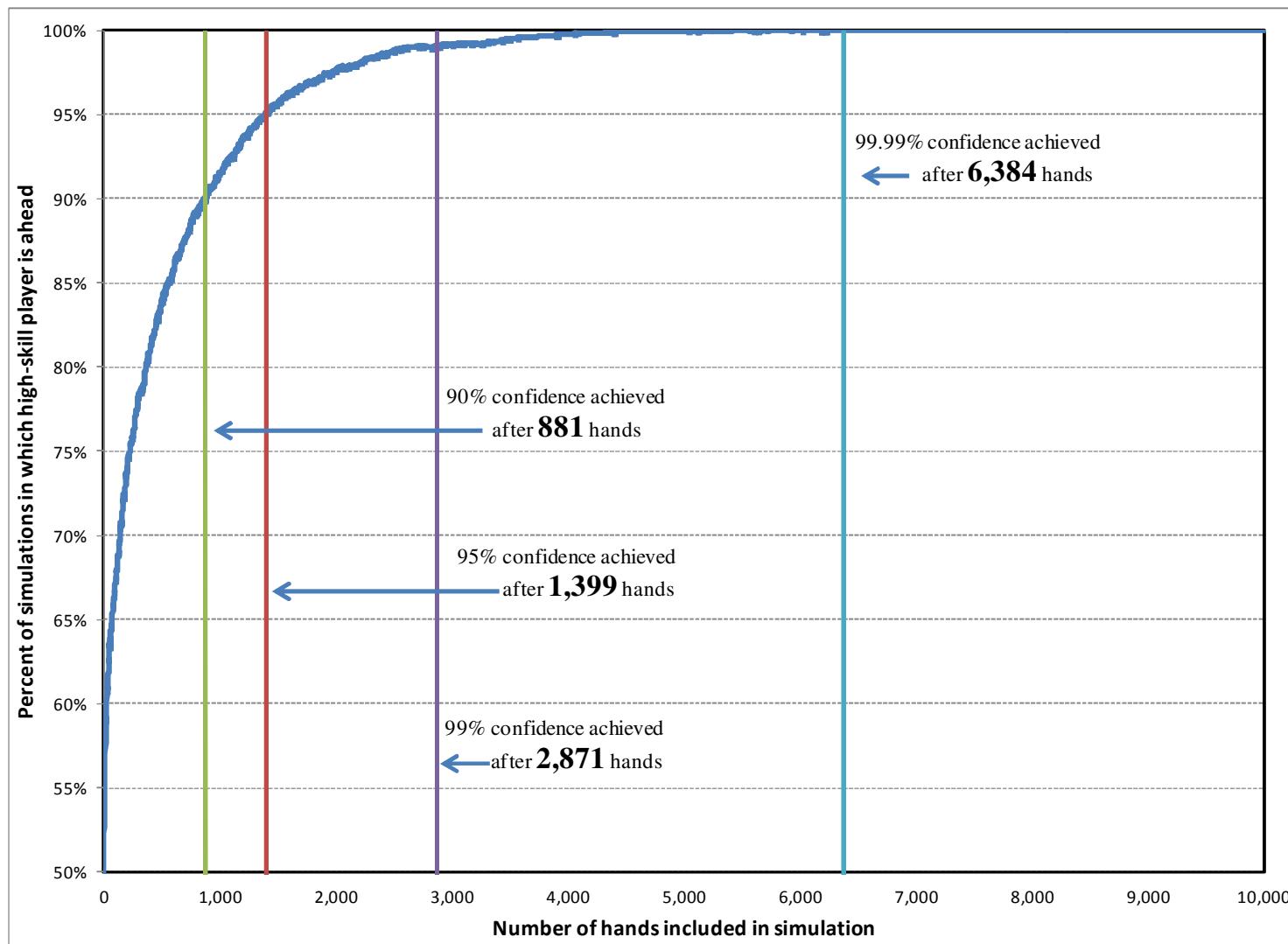


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- (75) I now turn to the actual results of my simulations, which are based on sorting the players into higher and lower skill based upon their skill index. Figure 16 below shows the results for \$1/\$2 stakes. Several features of Figure 16 are striking. First, there is a near immediate departure from the results that would obtain if poker were a game of chance. If poker were a game of chance the graph of the number of simulations in which the higher skilled player is ahead would be flat and would hover around 50%. In stark contrast, the number of simulations in which the higher skilled player is ahead slopes upward in a very steep fashion before eventually beginning to level off. After only 30 hands played, over 60% of the simulations show the higher skill group to be ahead of the lower skill group. After 225 hands played, nearly 75% of the simulations show the higher skill group to be ahead. This immediate and substantial departure from 50% provides a clear distinction between poker and a game of chance. Moreover, statisticians often use confidence levels of 90% and 95% when identifying a statistically significant finding. In this instance, after 881 hands the higher skill group is ahead in 90% of the simulations. Moreover, the more simulations I run, the more the dominance of the higher skill group becomes apparent. By 1,399 hands, the higher skill group is ahead in 95% of the simulations. By the time I get to 6,384 hands, the higher skill group is ahead in 99.99% of the simulations.

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Figure 16: Percentage of the time a higher skilled player (top 50% of skill) would predominate over a lower skilled player (bottom 50% of skill) after any given number of hands at \$1/\$2 stakes



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- (76) The results of these simulations provide clear and powerful evidence that poker differs significantly from a game of chance. Moreover, the number of hands by which the higher skilled players predominate with a high degree of certainty could be played in a few sessions of poker. I find similar results for all of the stakes, as shown in the Appendix, Section IV.E.
- (77) After 1,399 hands at \$1/\$2 stakes, the higher skilled players are ahead in over 95% of the simulations. I also asked the question, "How likely would the lower skilled players be to catch up if 1,399 more hands were to be played?" In the simulations in which the lower skilled players are behind after 1,399 hands, they are able to catch up in the next 1,399 hands only 0.003579 of the time (34 times out of 9,500 simulations). This further corroborates the conclusion that poker is a game in which skill predominates over chance. I find similar results for all of the stakes, as shown in the Appendix, Section IV.F.
- (78) My conclusions based on these results are robust to any imprecision in the measure of predicted skill. Any "noise" in the analysis, or any omitted factors that would cause skill to be imprecisely or incorrectly estimated would cause my analysis to suggest that skill is less important in poker than it really is. Thus my conclusion is conservative, because skill is even more important in poker than the analysis herein suggests. Suppose that an aspect of skill is neglected in the prediction of the skill index. Taking optimal account of stack sizes and proper sizing of bets are two enormously important aspects of poker skill that are not reflected in the above analyses, for example. Because of this, some players who are relatively better or worse at these skills compared to their other skills might be misclassified as less or more skillful, respectively, than they really are. This means that my lower skill group may actually contain some higher skilled players, and their higher win rates would be reducing the measured difference between the high and low skill groups. Similarly, having lower skilled players incorrectly identified as high skill means that their lower win rates are bringing down the measured performance of the high group, again reducing the measured returns to skill. If the skill index was simply random, then there would be no measurable difference between the high and low skill group, even if poker was in fact a game predominated by skill. Thus any noise in the creation of the skill index makes it harder to detect the importance of skill. The fact that I have detected enough skill to demonstrate that skill predominates over chance in poker, even with an imperfect measure of skill, means that skill is in fact even more predominant over chance than the measured results show.



Randal D. Heeb

July 5, 2012

Date

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IV. Appendix

IV.A. Summary statistics for all games

Figure 17: Summary statistics for top and bottom players as measured by total dollars won

Stakes	Top 50 winning players				Bottom 50 losing players			
	Average hands played	Average amount won	Winning months	Losing months	Average hands played	Average amount won	Winning months	Losing months
\$1/\$2	854,717	\$43,662	439	117	784,235	-\$31,574	85	437
\$2/\$4	453,953	\$57,433	434	109	255,542	-\$27,255	105	377
\$3/\$6	381,991	\$82,703	460	102	121,547	-\$28,729	104	358
\$5/\$10	292,945	\$97,922	410	142	62,053	-\$40,691	72	342
\$10/\$20	63,321	\$83,995	355	158	23,535	-\$50,695	102	296

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IV.B. Win rates for players with better and worse results on all other hands for each of 169 possible starting hands

Figure 18: Win rates for players with better and worse results on all other hands for each of 169 possible starting hands, for \$0.50/\$1.00 stakes

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
A A	\$9.44	\$8.45	\$0.06	✓
2 2	-\$0.14	-\$0.51	\$0.02	✓
3 3	-\$0.11	-\$0.47	\$0.03	✓
4 4	-\$0.01	-\$0.44	\$0.03	✓
5 5	\$0.06	-\$0.40	\$0.03	✓
6 6	\$0.16	-\$0.21	\$0.03	✓
7 7	\$0.29	-\$0.18	\$0.03	✓
8 8	\$0.53	\$0.12	\$0.04	✓
9 9	\$0.86	\$0.44	\$0.04	✓
T T	\$1.40	\$0.95	\$0.04	✓
J J	\$2.24	\$1.88	\$0.05	✓
Q Q	\$3.67	\$3.22	\$0.06	✓
K K	\$6.13	\$5.46	\$0.06	✓
K Q s	\$0.42	-\$0.03	\$0.04	✓
K J s	\$0.22	-\$0.16	\$0.04	✓
K T s	\$0.08	-\$0.31	\$0.03	✓
K 9 s	-\$0.10	-\$0.46	\$0.02	✓
K 8 s	-\$0.16	-\$0.46	\$0.02	✓
K 7 s	-\$0.18	-\$0.49	\$0.02	✓
K 6 s	-\$0.20	-\$0.45	\$0.02	✓
K 5 s	-\$0.20	-\$0.48	\$0.02	✓
K 4 s	-\$0.21	-\$0.45	\$0.01	✓
K 3 s	-\$0.22	-\$0.41	\$0.01	✓
K 2 s	-\$0.21	-\$0.49	\$0.01	✓
K A s	\$2.15	\$1.58	\$0.06	✓
Q J s	\$0.12	-\$0.33	\$0.04	✓
Q T s	\$0.03	-\$0.34	\$0.03	✓
Q 9 s	-\$0.14	-\$0.48	\$0.03	✓
Q 8 s	-\$0.19	-\$0.50	\$0.02	✓
Q 7 s	-\$0.21	-\$0.56	\$0.02	✓
Q 6 s	-\$0.21	-\$0.49	\$0.01	✓
Q 5 s	-\$0.22	-\$0.49	\$0.01	✓
Q 4 s	-\$0.22	-\$0.44	\$0.01	✓
Q 3 s	-\$0.22	-\$0.44	\$0.01	✓
Q 2 s	-\$0.23	-\$0.45	\$0.01	✓
Q A s	\$1.06	\$0.65	\$0.05	✓
J T s	\$0.00	-\$0.36	\$0.04	✓
J 9 s	-\$0.13	-\$0.51	\$0.03	✓
J 8 s	-\$0.20	-\$0.53	\$0.02	✓
J 7 s	-\$0.23	-\$0.55	\$0.02	✓
J 6 s	-\$0.22	-\$0.51	\$0.01	✓
J 5 s	-\$0.23	-\$0.47	\$0.01	✓
J 4 s	-\$0.23	-\$0.46	\$0.01	✓
J 3 s	-\$0.23	-\$0.42	\$0.01	✓
J 2 s	-\$0.23	-\$0.44	\$0.01	✓
J A s	\$0.59	\$0.19	\$0.04	✓
T 9 s	-\$0.08	-\$0.48	\$0.03	✓
T 8 s	-\$0.19	-\$0.52	\$0.03	✓
T 7 s	-\$0.23	-\$0.53	\$0.02	✓
T 6 s	-\$0.23	-\$0.45	\$0.01	✓
T 5 s	-\$0.24	-\$0.45	\$0.01	✓
T 4 s	-\$0.22	-\$0.42	\$0.01	✓
T 3 s	-\$0.23	-\$0.38	\$0.01	✓
T 2 s	-\$0.23	-\$0.39	\$0.01	✓
T A s	\$0.38	-\$0.04	\$0.04	✓
9 8 s	-\$0.17	-\$0.55	\$0.03	✓
9 7 s	-\$0.20	-\$0.52	\$0.03	✓
9 6 s	-\$0.25	-\$0.50	\$0.02	✓
9 5 s	-\$0.24	-\$0.47	\$0.01	✓
9 4 s	-\$0.23	-\$0.40	\$0.01	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
9 3 s	-\$0.23	-\$0.37	\$0.01	✓
9 2 s	-\$0.23	-\$0.34	\$0.01	✓
9 A s	\$0.07	-\$0.25	\$0.03	✓
8 7 s	-\$0.23	-\$0.61	\$0.03	✓
8 6 s	-\$0.24	-\$0.44	\$0.02	✓
8 5 s	-\$0.25	-\$0.52	\$0.02	✓
8 4 s	-\$0.24	-\$0.43	\$0.01	✓
8 3 s	-\$0.23	-\$0.40	\$0.01	✓
8 2 s	-\$0.23	-\$0.38	\$0.01	✓
8 A s	-\$0.02	-\$0.38	\$0.03	✓
7 6 s	-\$0.22	-\$0.50	\$0.03	✓
7 5 s	-\$0.25	-\$0.46	\$0.02	✓
7 4 s	-\$0.25	-\$0.48	\$0.01	✓
7 3 s	-\$0.23	-\$0.41	\$0.01	✓
7 2 s	-\$0.23	-\$0.37	\$0.01	✓
7 A s	-\$0.06	-\$0.55	\$0.03	✓
6 5 s	-\$0.26	-\$0.51	\$0.03	✓
6 4 s	-\$0.25	-\$0.47	\$0.02	✓
6 3 s	-\$0.24	-\$0.46	\$0.01	✓
6 2 s	-\$0.23	-\$0.39	\$0.01	✓
6 A s	-\$0.12	-\$0.54	\$0.03	✓
5 4 s	-\$0.25	-\$0.54	\$0.03	✓
5 3 s	-\$0.25	-\$0.47	\$0.02	✓
5 2 s	-\$0.24	-\$0.42	\$0.01	✓
5 A s	-\$0.06	-\$0.38	\$0.03	✓
4 3 s	-\$0.26	-\$0.55	\$0.02	✓
4 2 s	-\$0.26	-\$0.49	\$0.01	✓
4 A s	-\$0.10	-\$0.40	\$0.03	✓
3 2 s	-\$0.26	-\$0.56	\$0.01	✓
3 A s	-\$0.12	-\$0.51	\$0.03	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
2 A s	-\$0.15	-\$0.58	\$0.02	✓
K Q o	\$0.01	-\$0.47	\$0.02	✓
K J o	-\$0.12	-\$0.62	\$0.01	✓
K T o	-\$0.17	-\$0.62	\$0.01	✓
K 9 o	-\$0.21	-\$0.55	\$0.01	✓
K 8 o	-\$0.22	-\$0.45	\$0.01	✓
K 7 o	-\$0.22	-\$0.40	\$0.00	✓
K 6 o	-\$0.22	-\$0.37	\$0.00	✓
K 5 o	-\$0.22	-\$0.38	\$0.00	✓
K 4 o	-\$0.21	-\$0.34	\$0.00	✓
K 3 o	-\$0.22	-\$0.33	\$0.00	✓
K 2 o	-\$0.22	-\$0.33	\$0.00	✓
K A o	\$1.40	\$0.97	\$0.03	✓
Q J o	-\$0.18	-\$0.64	\$0.01	✓
Q T o	-\$0.19	-\$0.63	\$0.01	✓
Q 9 o	-\$0.22	-\$0.53	\$0.01	✓
Q 8 o	-\$0.22	-\$0.41	\$0.00	✓
Q 7 o	-\$0.22	-\$0.35	\$0.00	✓
Q 6 o	-\$0.22	-\$0.32	\$0.00	✓
Q 5 o	-\$0.22	-\$0.33	\$0.00	✓
Q 4 o	-\$0.22	-\$0.30	\$0.00	✓
Q 3 o	-\$0.22	-\$0.29	\$0.00	✓
Q 2 o	-\$0.22	-\$0.28	\$0.00	✓
Q A o	\$0.47	-\$0.03	\$0.02	✓
J T o	-\$0.21	-\$0.66	\$0.01	✓
J 9 o	-\$0.22	-\$0.53	\$0.01	✓
J 8 o	-\$0.22	-\$0.46	\$0.01	✓
J 7 o	-\$0.22	-\$0.34	\$0.00	✓
J 6 o	-\$0.22	-\$0.30	\$0.00	✓
J 5 o	-\$0.22	-\$0.30	\$0.00	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
J 4 o	-\$0.22	-\$0.28	\$0.00	✓
J 3 o	-\$0.22	-\$0.28	\$0.00	✓
J 2 o	-\$0.22	-\$0.27	\$0.00	✓
J A o	\$0.11	-\$0.41	\$0.02	✓
T 9 o	-\$0.22	-\$0.54	\$0.01	✓
T 8 o	-\$0.22	-\$0.41	\$0.01	✓
T 7 o	-\$0.22	-\$0.35	\$0.00	✓
T 6 o	-\$0.22	-\$0.30	\$0.00	✓
T 5 o	-\$0.22	-\$0.28	\$0.00	✓
T 4 o	-\$0.22	-\$0.27	\$0.00	✓
T 3 o	-\$0.22	-\$0.26	\$0.00	✓
T 2 o	-\$0.22	-\$0.27	\$0.00	✓
T A o	-\$0.05	-\$0.58	\$0.02	✓
9 8 o	-\$0.24	-\$0.54	\$0.01	✓
9 7 o	-\$0.23	-\$0.38	\$0.00	✓
9 6 o	-\$0.22	-\$0.32	\$0.00	✓
9 5 o	-\$0.22	-\$0.28	\$0.00	✓
9 4 o	-\$0.22	-\$0.26	\$0.00	✓
9 3 o	-\$0.22	-\$0.26	\$0.00	✓
9 2 o	-\$0.22	-\$0.26	\$0.00	✓
9 A o	-\$0.17	-\$0.59	\$0.01	✓
8 7 o	-\$0.24	-\$0.49	\$0.01	✓
8 6 o	-\$0.23	-\$0.35	\$0.00	✓
8 5 o	-\$0.22	-\$0.30	\$0.00	✓
8 4 o	-\$0.22	-\$0.27	\$0.00	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
8 3 o	-\$0.22	-\$0.26	\$0.00	✓
8 2 o	-\$0.22	-\$0.25	\$0.00	✓
8 A o	-\$0.20	-\$0.65	\$0.01	✓
7 6 o	-\$0.23	-\$0.43	\$0.01	✓
7 5 o	-\$0.22	-\$0.33	\$0.00	✓
7 4 o	-\$0.22	-\$0.29	\$0.00	✓
7 3 o	-\$0.22	-\$0.26	\$0.00	✓
7 2 o	-\$0.23	-\$0.26	\$0.00	✓
7 A o	-\$0.21	-\$0.62	\$0.01	✓
6 5 o	-\$0.23	-\$0.40	\$0.01	✓
6 4 o	-\$0.22	-\$0.31	\$0.00	✓
6 3 o	-\$0.22	-\$0.28	\$0.00	✓
6 2 o	-\$0.22	-\$0.27	\$0.00	✓
6 A o	-\$0.22	-\$0.59	\$0.01	✓
5 4 o	-\$0.23	-\$0.34	\$0.00	✓
5 3 o	-\$0.22	-\$0.30	\$0.00	✓
5 2 o	-\$0.22	-\$0.29	\$0.00	✓
5 A o	-\$0.22	-\$0.60	\$0.01	✓
4 3 o	-\$0.23	-\$0.32	\$0.00	✓
4 2 o	-\$0.23	-\$0.29	\$0.00	✓
4 A o	-\$0.21	-\$0.57	\$0.01	✓
3 2 o	-\$0.23	-\$0.32	\$0.00	✓
3 A o	-\$0.22	-\$0.53	\$0.01	✓
2 A o	-\$0.23	-\$0.57	\$0.01	✓

of 169 starting hands in which players with *better* results on all other hands have statistically significant higher win rates: 169# of 169 starting hands in which players with *worse* results on all other hands have statistically significant higher win rates: 0

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Figure 19: Win rates for players with better and worse results on all other hands for each of 169 possible starting hands, for \$2/\$4 stakes

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
A A	\$35.71	\$33.00	\$0.39	✓
2 2	-\$0.40	-\$0.95	\$0.16	✓
3 3	-\$0.39	-\$1.15	\$0.16	✓
4 4	\$0.14	-\$1.00	\$0.17	✓
5 5	\$0.34	-\$0.59	\$0.18	✓
6 6	\$0.84	-\$0.28	\$0.19	✓
7 7	\$1.46	\$1.04	\$0.21	
8 8	\$2.55	\$1.02	\$0.23	✓
9 9	\$3.76	\$2.60	\$0.26	✓
TT	\$6.30	\$4.67	\$0.30	✓
JJ	\$9.85	\$8.42	\$0.34	✓
QQ	\$15.61	\$13.53	\$0.38	✓
KK	\$24.49	\$20.85	\$0.39	✓
K Q s	\$2.09	\$0.64	\$0.27	✓
K J s	\$1.03	-\$0.16	\$0.24	✓
K T s	\$0.62	-\$0.30	\$0.21	✓
K 9 s	-\$0.36	-\$1.15	\$0.16	✓
K 8 s	-\$0.52	-\$1.60	\$0.12	✓
K 7 s	-\$0.65	-\$1.17	\$0.11	✓
K 6 s	-\$0.73	-\$1.18	\$0.10	✓
K 5 s	-\$0.77	-\$1.10	\$0.10	✓
K 4 s	-\$0.70	-\$1.59	\$0.09	✓
K 3 s	-\$0.76	-\$1.45	\$0.09	✓
K 2 s	-\$0.79	-\$1.61	\$0.08	✓
K A s	\$9.20	\$7.44	\$0.38	✓
Q J s	\$0.73	-\$0.84	\$0.23	✓
Q T s	\$0.46	-\$0.58	\$0.21	✓
Q 9 s	-\$0.44	-\$1.23	\$0.16	✓
Q 8 s	-\$0.73	-\$1.70	\$0.12	✓
Q 7 s	-\$0.84	-\$1.53	\$0.10	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
Q 6 s	-\$0.83	-\$1.68	\$0.08	✓
Q 5 s	-\$0.80	-\$1.43	\$0.08	✓
Q 4 s	-\$0.82	-\$1.32	\$0.07	✓
Q 3 s	-\$0.79	-\$1.44	\$0.07	✓
Q 2 s	-\$0.86	-\$1.24	\$0.06	✓
Q A s	\$4.76	\$3.72	\$0.32	✓
J T s	\$0.40	-\$0.99	\$0.23	✓
J 9 s	-\$0.39	-\$1.06	\$0.18	✓
J 8 s	-\$0.68	-\$1.58	\$0.14	✓
J 7 s	-\$0.87	-\$1.79	\$0.10	✓
J 6 s	-\$0.86	-\$1.32	\$0.07	✓
J 5 s	-\$0.85	-\$1.41	\$0.07	✓
J 4 s	-\$0.88	-\$1.15	\$0.06	✓
J 3 s	-\$0.86	-\$1.43	\$0.05	✓
J 2 s	-\$0.87	-\$1.23	\$0.05	✓
J A s	\$2.79	\$2.45	\$0.27	
T 9 s	-\$0.13	-\$1.36	\$0.22	✓
T 8 s	-\$0.62	-\$1.83	\$0.17	✓
T 7 s	-\$0.86	-\$1.45	\$0.12	✓
T 6 s	-\$0.81	-\$1.57	\$0.08	✓
T 5 s	-\$0.91	-\$1.49	\$0.06	✓
T 4 s	-\$0.85	-\$1.36	\$0.05	✓
T 3 s	-\$0.87	-\$1.27	\$0.05	✓
T 2 s	-\$0.87	-\$1.27	\$0.04	✓
T A s	\$1.89	\$0.71	\$0.24	✓
9 8 s	-\$0.54	-\$1.48	\$0.21	✓
9 7 s	-\$0.61	-\$1.59	\$0.16	✓
9 6 s	-\$0.99	-\$1.74	\$0.11	✓
9 5 s	-\$0.87	-\$1.59	\$0.07	✓
9 4 s	-\$0.90	-\$1.32	\$0.05	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
9 3 s	-\$0.87	-\$1.27	\$0.04	✓
9 2 s	-\$0.88	-\$1.17	\$0.04	✓
9 A s	\$0.79	-\$0.39	\$0.20	✓
8 7 s	-\$0.71	-\$1.83	\$0.19	✓
8 6 s	-\$0.79	-\$1.71	\$0.15	✓
8 5 s	-\$0.91	-\$1.71	\$0.10	✓
8 4 s	-\$0.94	-\$1.45	\$0.07	✓
8 3 s	-\$0.88	-\$1.37	\$0.04	✓
8 2 s	-\$0.87	-\$1.44	\$0.04	✓
8 A s	\$0.18	-\$1.46	\$0.18	✓
7 6 s	-\$0.67	-\$1.26	\$0.18	✓
7 5 s	-\$0.80	-\$1.63	\$0.14	✓
7 4 s	-\$0.89	-\$1.71	\$0.09	✓
7 3 s	-\$0.92	-\$1.62	\$0.05	✓
7 2 s	-\$0.89	-\$1.36	\$0.04	✓
7 A s	\$0.13	-\$1.35	\$0.17	✓
6 5 s	-\$0.82	-\$1.53	\$0.17	✓
6 4 s	-\$0.85	-\$1.28	\$0.12	✓
6 3 s	-\$0.94	-\$1.24	\$0.08	✓
6 2 s	-\$0.91	-\$1.31	\$0.05	✓
6 A s	-\$0.30	-\$1.73	\$0.16	✓
5 4 s	-\$0.75	-\$1.51	\$0.16	✓
5 3 s	-\$0.91	-\$1.18	\$0.10	✓
5 2 s	-\$0.89	-\$1.41	\$0.06	✓
5 A s	-\$0.12	-\$0.72	\$0.18	✓
4 3 s	-\$0.97	-\$1.40	\$0.11	✓
4 2 s	-\$0.90	-\$1.51	\$0.07	✓
4 A s	-\$0.25	-\$1.16	\$0.17	✓
3 2 s	-\$0.96	-\$1.53	\$0.07	✓
3 A s	-\$0.45	-\$1.31	\$0.17	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
2 A s	-\$0.47	-\$1.59	\$0.16	✓
K Q o	\$0.33	-\$0.70	\$0.12	✓
K J o	-\$0.24	-\$1.40	\$0.09	✓
K T o	-\$0.57	-\$1.62	\$0.07	✓
K 9 o	-\$0.79	-\$1.43	\$0.04	✓
K 8 o	-\$0.82	-\$1.62	\$0.03	✓
K 7 o	-\$0.85	-\$1.32	\$0.03	✓
K 6 o	-\$0.85	-\$1.20	\$0.02	✓
K 5 o	-\$0.86	-\$1.17	\$0.02	✓
K 4 o	-\$0.85	-\$1.15	\$0.02	✓
K 3 o	-\$0.87	-\$1.13	\$0.02	✓
K 2 o	-\$0.87	-\$1.10	\$0.02	✓
K A o	\$6.76	\$5.04	\$0.21	✓
Q J o	-\$0.52	-\$1.70	\$0.08	✓
Q T o	-\$0.64	-\$2.05	\$0.07	✓
Q 9 o	-\$0.81	-\$1.63	\$0.04	✓
Q 8 o	-\$0.84	-\$1.28	\$0.03	✓
Q 7 o	-\$0.86	-\$1.16	\$0.02	✓
Q 6 o	-\$0.86	-\$1.05	\$0.02	✓
Q 5 o	-\$0.86	-\$1.08	\$0.02	✓
Q 4 o	-\$0.87	-\$1.14	\$0.01	✓
Q 3 o	-\$0.87	-\$0.99	\$0.01	✓
Q 2 o	-\$0.88	-\$1.06	\$0.01	✓
Q A o	\$2.72	\$0.93	\$0.16	✓
J T o	-\$0.75	-\$2.20	\$0.07	✓
J 9 o	-\$0.83	-\$1.45	\$0.04	✓
J 8 o	-\$0.89	-\$1.33	\$0.03	✓
J 7 o	-\$0.87	-\$1.12	\$0.02	✓
J 6 o	-\$0.87	-\$1.06	\$0.01	✓
J 5 o	-\$0.87	-\$1.01	\$0.01	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
J 4 o	-\$0.87	-\$1.07	\$0.01	✓
J 3 o	-\$0.87	-\$1.02	\$0.01	✓
J 2 o	-\$0.88	-\$1.01	\$0.01	✓
J A o	\$0.90	-\$0.89	\$0.12	✓
T 9 o	-\$0.84	-\$1.67	\$0.05	✓
T 8 o	-\$0.84	-\$1.32	\$0.03	✓
T 7 o	-\$0.88	-\$1.16	\$0.02	✓
T 6 o	-\$0.88	-\$1.04	\$0.02	✓
T 5 o	-\$0.87	-\$1.03	\$0.01	✓
T 4 o	-\$0.87	-\$1.02	\$0.01	✓
T 3 o	-\$0.89	-\$0.91	\$0.01	✓
T 2 o	-\$0.88	-\$1.01	\$0.01	✓
T A o	\$0.17	-\$1.47	\$0.10	✓
9 8 o	-\$0.86	-\$1.75	\$0.04	✓
9 7 o	-\$0.85	-\$1.31	\$0.03	✓
9 6 o	-\$0.88	-\$1.19	\$0.02	✓
9 5 o	-\$0.88	-\$1.03	\$0.01	✓
9 4 o	-\$0.88	-\$0.96	\$0.01	✓
9 3 o	-\$0.88	-\$0.99	\$0.01	✓
9 2 o	-\$0.89	-\$0.97	\$0.01	✓
9 A o	-\$0.46	-\$1.99	\$0.07	✓
8 7 o	-\$0.87	-\$1.73	\$0.04	✓
8 6 o	-\$0.89	-\$1.24	\$0.02	✓
8 5 o	-\$0.88	-\$1.04	\$0.02	✓
8 4 o	-\$0.89	-\$1.02	\$0.01	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
8 3 o	-\$0.89	-\$0.98	\$0.01	✓
8 2 o	-\$0.89	-\$0.96	\$0.01	✓
8 A o	-\$0.58	-\$1.95	\$0.05	✓
7 6 o	-\$0.88	-\$1.35	\$0.03	✓
7 5 o	-\$0.89	-\$1.12	\$0.02	✓
7 4 o	-\$0.88	-\$1.07	\$0.02	✓
7 3 o	-\$0.89	-\$1.00	\$0.01	✓
7 2 o	-\$0.89	-\$0.98	\$0.01	✓
7 A o	-\$0.74	-\$1.91	\$0.05	✓
6 5 o	-\$0.91	-\$1.36	\$0.03	✓
6 4 o	-\$0.88	-\$1.12	\$0.02	✓
6 3 o	-\$0.88	-\$1.02	\$0.01	✓
6 2 o	-\$0.89	-\$1.00	\$0.01	✓
6 A o	-\$0.78	-\$1.74	\$0.04	✓
5 4 o	-\$0.90	-\$1.30	\$0.02	✓
5 3 o	-\$0.89	-\$1.06	\$0.02	✓
5 2 o	-\$0.89	-\$1.01	\$0.01	✓
5 A o	-\$0.77	-\$1.60	\$0.04	✓
4 3 o	-\$0.90	-\$1.21	\$0.02	✓
4 2 o	-\$0.90	-\$1.00	\$0.01	✓
4 A o	-\$0.78	-\$1.60	\$0.04	✓
3 2 o	-\$0.90	-\$1.13	\$0.01	✓
3 A o	-\$0.81	-\$1.58	\$0.04	✓
2 A o	-\$0.83	-\$1.64	\$0.03	✓

of 169 starting hands in which players with *better* results on all other hands have statistically significant higher win rates: 167# of 169 starting hands in which players with *worse* results on all other hands have statistically significant higher win rates: 0

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Figure 20: Win rates for players with better and worse results on all other hands for each of 169 possible starting hands, for \$3/\$6 stakes

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
A A	\$56.85	\$51.48	\$0.94	✓
2 2	-\$0.65	-\$2.35	\$0.37	✓
3 3	-\$0.22	-\$1.93	\$0.38	✓
4 4	\$0.27	-\$1.29	\$0.40	✓
5 5	\$0.92	-\$1.50	\$0.44	✓
6 6	\$1.90	\$0.62	\$0.46	✓
7 7	\$2.52	\$0.34	\$0.50	✓
8 8	\$3.98	\$1.39	\$0.55	✓
9 9	\$5.88	\$4.00	\$0.61	✓
TT	\$10.30	\$6.52	\$0.70	✓
JJ	\$15.25	\$14.04	\$0.80	
QQ	\$23.17	\$21.34	\$0.90	✓
KK	\$37.48	\$33.77	\$0.93	✓
K Q s	\$3.38	\$0.59	\$0.64	✓
K J s	\$2.34	-\$0.93	\$0.57	✓
K T s	\$1.38	-\$0.26	\$0.51	✓
K 9 s	-\$0.40	-\$1.54	\$0.37	✓
K 8 s	-\$0.96	-\$3.06	\$0.31	✓
K 7 s	-\$0.77	-\$1.89	\$0.27	✓
K 6 s	-\$1.01	-\$1.76	\$0.25	✓
K 5 s	-\$1.08	-\$2.27	\$0.24	✓
K 4 s	-\$1.00	-\$2.20	\$0.23	✓
K 3 s	-\$1.06	-\$2.47	\$0.22	✓
K 2 s	-\$1.03	-\$1.79	\$0.20	✓
K A s	\$13.71	\$11.85	\$0.91	✓
Q J s	\$1.46	-\$0.83	\$0.57	✓
Q T s	\$0.29	-\$1.22	\$0.51	✓
Q 9 s	-\$0.40	-\$0.44	\$0.39	
Q 8 s	-\$1.22	-\$2.49	\$0.30	✓
Q 7 s	-\$1.15	-\$1.58	\$0.22	

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
Q 6 s	-\$1.15	-\$2.19	\$0.20	✓
Q 5 s	-\$1.27	-\$1.87	\$0.19	✓
Q 4 s	-\$1.19	-\$1.98	\$0.18	✓
Q 3 s	-\$1.29	-\$2.33	\$0.15	✓
Q 2 s	-\$1.28	-\$2.04	\$0.15	✓
Q A s	\$7.39	\$5.41	\$0.78	✓
J T s	\$0.98	-\$1.68	\$0.56	✓
J 9 s	-\$0.77	-\$2.81	\$0.45	✓
J 8 s	-\$0.80	-\$1.77	\$0.34	✓
J 7 s	\$1.00	-\$2.27	\$0.23	✓
J 6 s	\$1.35	-\$2.12	\$0.17	✓
J 5 s	\$1.25	-\$1.80	\$0.16	✓
J 4 s	\$1.31	-\$1.92	\$0.15	✓
J 3 s	\$1.27	-\$2.25	\$0.15	✓
J 2 s	\$1.45	-\$2.17	\$0.13	✓
J A s	\$4.69	\$2.90	\$0.64	✓
T 9 s	-\$0.12	-\$2.03	\$0.53	✓
T 8 s	-\$0.84	-\$1.71	\$0.41	✓
T 7 s	-\$1.09	-\$2.57	\$0.29	✓
T 6 s	-\$1.28	-\$2.77	\$0.20	✓
T 5 s	-\$1.37	-\$2.33	\$0.15	✓
T 4 s	-\$1.22	-\$2.12	\$0.13	✓
T 3 s	-\$1.35	-\$2.03	\$0.12	✓
T 2 s	-\$1.35	-\$1.89	\$0.12	✓
T A s	\$3.31	\$1.63	\$0.58	✓
9 8 s	-\$0.89	-\$1.57	\$0.49	
9 7 s	-\$0.98	-\$1.83	\$0.40	✓
9 6 s	-\$1.17	-\$2.21	\$0.27	✓
9 5 s	-\$1.34	-\$1.95	\$0.17	✓
9 4 s	-\$1.32	-\$2.43	\$0.12	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
9 3 s	-\$1.36	-\$1.67	\$0.11	✓
9 2 s	-\$1.37	-\$1.74	\$0.10	✓
9 A s	\$0.91	-\$1.56	\$0.46	✓
8 7 s	-\$0.78	-\$1.80	\$0.48	✓
8 6 s	-\$1.26	-\$2.05	\$0.37	✓
8 5 s	-\$1.33	-\$3.15	\$0.25	✓
8 4 s	-\$1.33	-\$2.33	\$0.17	✓
8 3 s	-\$1.35	-\$1.93	\$0.10	✓
8 2 s	-\$1.40	-\$1.71	\$0.10	✓
8 A s	\$0.30	-\$2.20	\$0.45	✓
7 6 s	-\$0.80	-\$2.72	\$0.45	✓
7 5 s	-\$1.20	-\$1.69	\$0.34	
7 4 s	-\$1.31	-\$2.36	\$0.22	✓
7 3 s	-\$1.33	-\$1.50	\$0.13	
7 2 s	-\$1.31	-\$2.01	\$0.10	✓
7 A s	\$0.25	-\$1.55	\$0.42	✓
6 5 s	-\$0.88	-\$2.51	\$0.41	✓
6 4 s	-\$1.25	-\$2.51	\$0.30	✓
6 3 s	-\$1.43	-\$3.19	\$0.19	✓
6 2 s	-\$1.38	-\$2.15	\$0.11	✓
6 A s	-\$0.20	-\$1.36	\$0.40	✓
5 4 s	-\$1.18	-\$2.20	\$0.38	✓
5 3 s	-\$1.48	-\$2.41	\$0.24	✓
5 2 s	-\$1.39	-\$2.13	\$0.16	✓
5 A s	-\$0.23	-\$2.16	\$0.43	✓
4 3 s	-\$1.33	-\$3.12	\$0.27	✓
4 2 s	-\$1.37	-\$2.57	\$0.16	✓
4 A s	-\$0.16	-\$1.59	\$0.42	✓
3 2 s	-\$1.38	-\$2.58	\$0.18	✓
3 A s	-\$0.39	-\$1.52	\$0.40	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
2 A s	-\$0.55	-\$1.45	\$0.39	✓
K Q o	\$0.63	-\$1.20	\$0.28	✓
K J o	-\$0.13	-\$2.88	\$0.22	✓
K T o	-\$0.75	-\$2.55	\$0.17	✓
K 9 o	-\$1.22	-\$2.18	\$0.11	✓
K 8 o	-\$1.24	-\$2.26	\$0.07	✓
K 7 o	-\$1.32	-\$1.92	\$0.06	✓
K 6 o	-\$1.30	-\$1.85	\$0.05	✓
K 5 o	-\$1.31	-\$1.64	\$0.05	✓
K 4 o	-\$1.33	-\$1.71	\$0.04	✓
K 3 o	-\$1.34	-\$1.76	\$0.04	✓
K 2 o	-\$1.33	-\$1.50	\$0.04	✓
K A o	\$10.19	\$8.03	\$0.49	✓
Q J o	-\$0.67	-\$3.59	\$0.19	✓
Q T o	-\$0.88	-\$3.02	\$0.16	✓
Q 9 o	-\$1.22	-\$2.56	\$0.10	✓
Q 8 o	-\$1.28	-\$1.78	\$0.06	✓
Q 7 o	-\$1.33	-\$1.93	\$0.05	✓
Q 6 o	-\$1.32	-\$1.56	\$0.04	✓
Q 5 o	-\$1.32	-\$1.76	\$0.04	✓
Q 4 o	-\$1.31	-\$1.52	\$0.03	✓
Q 3 o	-\$1.33	-\$1.46	\$0.03	✓
Q 2 o	-\$1.35	-\$1.50	\$0.03	✓
Q A o	\$4.21	\$2.08	\$0.38	✓
J T o	-\$1.01	-\$3.14	\$0.17	✓
J 9 o	-\$1.22	-\$2.41	\$0.10	✓
J 8 o	-\$1.30	-\$2.05	\$0.07	✓
J 7 o	-\$1.33	-\$1.75	\$0.05	✓
J 6 o	-\$1.33	-\$1.68	\$0.03	✓
J 5 o	-\$1.33	-\$1.51	\$0.03	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
J 4 o	-\$1.33	-\$1.57	\$0.03	✓
J 3 o	-\$1.34	-\$1.40	\$0.03	✓
J 2 o	-\$1.34	-\$1.52	\$0.03	✓
J A o	\$1.80	-\$0.36	\$0.29	✓
T 9 o	-\$1.29	-\$2.42	\$0.12	✓
T 8 o	-\$1.32	-\$2.00	\$0.08	✓
T 7 o	-\$1.28	-\$1.89	\$0.06	✓
T 6 o	-\$1.34	-\$1.56	\$0.04	✓
T 5 o	-\$1.32	-\$1.59	\$0.03	✓
T 4 o	-\$1.32	-\$1.53	\$0.03	✓
T 3 o	-\$1.34	-\$1.45	\$0.02	✓
T 2 o	-\$1.33	-\$1.48	\$0.02	✓
T A o	\$0.45	-\$1.95	\$0.23	✓
9 8 o	-\$1.34	-\$2.61	\$0.11	✓
9 7 o	-\$1.37	-\$1.91	\$0.07	✓
9 6 o	-\$1.33	-\$1.77	\$0.05	✓
9 5 o	-\$1.34	-\$1.58	\$0.03	✓
9 4 o	-\$1.35	-\$1.46	\$0.03	✓
9 3 o	-\$1.35	-\$1.41	\$0.02	✓
9 2 o	-\$1.35	-\$1.38	\$0.02	
9 A o	-\$0.66	-\$2.84	\$0.15	✓
8 7 o	-\$1.34	-\$2.42	\$0.09	✓
8 6 o	-\$1.32	-\$1.95	\$0.06	✓
8 5 o	-\$1.33	-\$1.79	\$0.04	✓
8 4 o	-\$1.35	-\$1.58	\$0.03	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
8 3 o	-\$1.34	-\$1.39	\$0.02	
8 2 o	-\$1.34	-\$1.48	\$0.02	✓
8 A o	-\$0.90	-\$2.41	\$0.13	✓
7 6 o	-\$1.32	-\$2.16	\$0.08	✓
7 5 o	-\$1.33	-\$1.77	\$0.05	✓
7 4 o	-\$1.36	-\$1.67	\$0.04	✓
7 3 o	-\$1.34	-\$1.52	\$0.03	✓
7 2 o	-\$1.34	-\$1.49	\$0.02	✓
7 A o	-\$1.07	-\$3.13	\$0.12	✓
6 5 o	-\$1.33	-\$1.81	\$0.08	✓
6 4 o	-\$1.35	-\$1.74	\$0.05	✓
6 3 o	-\$1.35	-\$1.59	\$0.03	✓
6 2 o	-\$1.35	-\$1.51	\$0.02	✓
6 A o	-\$1.12	-\$2.70	\$0.10	✓
5 4 o	-\$1.38	-\$2.02	\$0.06	✓
5 3 o	-\$1.34	-\$1.78	\$0.04	✓
5 2 o	-\$1.38	-\$1.56	\$0.03	✓
5 A o	-\$1.16	-\$2.34	\$0.10	✓
4 3 o	-\$1.36	-\$2.00	\$0.04	✓
4 2 o	-\$1.36	-\$1.67	\$0.03	✓
4 A o	-\$1.23	-\$2.15	\$0.10	✓
3 2 o	-\$1.38	-\$1.65	\$0.04	✓
3 A o	-\$1.32	-\$2.10	\$0.09	✓
2 A o	-\$1.24	-\$2.52	\$0.08	✓

of 169 starting hands in which players with *better* results on all other hands have statistically significant higher win rates: 161# of 169 starting hands in which players with *worse* results on all other hands have statistically significant higher win rates: 0

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Figure 21: Win rates for players with better and worse results on all other hands for each of 169 possible starting hands, for \$5/\$10 stakes

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
A A	\$93.35	\$92.28	\$1.61	
2 2	-\$1.15	-\$3.02	\$0.62	✓
3 3	-\$0.28	-\$4.15	\$0.66	✓
4 4	\$0.28	-\$1.76	\$0.70	✓
5 5	\$0.80	-\$1.03	\$0.74	✓
6 6	\$2.64	\$1.66	\$0.80	
7 7	\$4.15	\$0.68	\$0.87	✓
8 8	\$6.76	\$3.71	\$0.97	✓
9 9	\$10.03	\$8.62	\$1.08	
TT	\$17.11	\$13.03	\$1.23	✓
J J	\$26.53	\$23.00	\$1.40	✓
Q Q	\$41.05	\$35.42	\$1.55	✓
K K	\$64.71	\$55.97	\$1.61	✓
K Q s	\$5.75	\$1.59	\$1.13	✓
K J s	\$3.32	\$1.03	\$1.02	✓
K T s	\$1.47	\$1.43	\$0.89	
K 9 s	-\$0.42	-\$2.61	\$0.67	✓
K 8 s	-\$1.44	-\$2.04	\$0.53	
K 7 s	-\$1.57	-\$3.40	\$0.48	✓
K 6 s	-\$1.57	-\$3.56	\$0.42	✓
K 5 s	-\$1.88	-\$3.65	\$0.40	✓
K 4 s	-\$1.60	-\$3.12	\$0.39	✓
K 3 s	-\$2.09	-\$3.06	\$0.37	✓
K 2 s	-\$2.02	-\$2.95	\$0.34	✓
K A s	\$24.04	\$24.10	\$1.59	
Q J s	\$2.03	-\$1.14	\$0.96	✓
Q T s	\$0.90	-\$0.98	\$0.90	✓
Q 9 s	-\$1.10	-\$2.06	\$0.68	
Q 8 s	-\$1.61	-\$2.61	\$0.50	✓
Q 7 s	-\$1.72	-\$3.73	\$0.38	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
Q 6 s	-\$2.16	-\$3.16	\$0.35	✓
Q 5 s	-\$2.09	-\$4.13	\$0.31	✓
Q 4 s	-\$2.14	-\$2.73	\$0.30	✓
Q 3 s	-\$2.06	-\$2.83	\$0.28	✓
Q 2 s	-\$2.02	-\$2.79	\$0.25	✓
Q A s	\$13.33	\$8.85	\$1.35	✓
J T s	\$0.29	\$0.25	\$0.97	
J 9 s	\$0.07	-\$1.94	\$0.79	✓
J 8 s	-\$1.63	-\$4.04	\$0.58	✓
J 7 s	-\$2.01	-\$3.09	\$0.41	✓
J 6 s	-\$2.10	-\$3.32	\$0.28	✓
J 5 s	-\$2.09	-\$3.03	\$0.27	✓
J 4 s	-\$2.12	-\$2.64	\$0.24	✓
J 3 s	-\$2.29	-\$3.08	\$0.21	✓
J 2 s	-\$2.11	-\$3.37	\$0.22	✓
J A s	\$7.72	\$5.74	\$1.14	
T 9 s	-\$0.12	-\$2.62	\$0.91	✓
T 8 s	-\$1.18	-\$0.96	\$0.73	
T 7 s	-\$1.75	-\$1.65	\$0.53	
T 6 s	-\$1.96	-\$3.22	\$0.35	✓
T 5 s	-\$2.24	-\$2.93	\$0.24	✓
T 4 s	-\$2.15	-\$4.30	\$0.22	✓
T 3 s	-\$2.30	-\$3.17	\$0.21	✓
T 2 s	-\$2.09	-\$2.85	\$0.18	✓
T A s	\$3.78	\$1.60	\$1.01	✓
9 8 s	-\$1.60	-\$1.91	\$0.88	
9 7 s	-\$1.42	-\$3.53	\$0.69	✓
9 6 s	-\$2.00	-\$3.26	\$0.48	✓
9 5 s	-\$2.19	-\$4.27	\$0.29	✓
9 4 s	-\$2.24	-\$3.61	\$0.19	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
9 3 s	-\$2.25	-\$2.53	\$0.19	
9 2 s	-\$2.32	-\$2.35	\$0.17	
9 A s	\$1.55	-\$2.21	\$0.85	✓
8 7 s	-\$1.26	-\$5.04	\$0.84	✓
8 6 s	-\$1.38	-\$3.64	\$0.64	✓
8 5 s	-\$2.45	-\$1.98	\$0.43	
8 4 s	-\$2.30	-\$2.56	\$0.26	
8 3 s	-\$2.19	-\$2.88	\$0.16	✓
8 2 s	-\$2.17	-\$2.80	\$0.15	✓
8 A s	\$0.82	-\$2.08	\$0.78	✓
7 6 s	-\$1.85	-\$2.97	\$0.81	
7 5 s	-\$2.01	-\$2.88	\$0.59	
7 4 s	-\$2.18	-\$3.86	\$0.38	✓
7 3 s	-\$2.10	-\$2.59	\$0.23	✓
7 2 s	-\$2.35	-\$2.31	\$0.18	
7 A s	\$0.42	-\$2.97	\$0.74	✓
6 5 s	-\$2.21	-\$2.06	\$0.74	
6 4 s	-\$2.12	-\$4.02	\$0.54	✓
6 3 s	-\$2.38	-\$3.54	\$0.34	✓
6 2 s	-\$2.21	-\$2.95	\$0.20	✓
6 A s	-\$0.55	-\$2.40	\$0.69	✓
5 4 s	-\$1.61	-\$1.82	\$0.68	
5 3 s	-\$1.95	-\$3.87	\$0.45	✓
5 2 s	-\$2.22	-\$2.33	\$0.27	
5 A s	\$0.11	-\$1.50	\$0.74	✓
4 3 s	-\$2.45	-\$3.88	\$0.47	✓
4 2 s	-\$2.34	-\$3.44	\$0.29	✓
4 A s	\$0.02	-\$2.66	\$0.72	✓
3 2 s	-\$2.39	-\$3.15	\$0.29	✓
3 A s	-\$0.44	-\$0.22	\$0.70	

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
2 A s	-\$1.01	-\$2.53	\$0.67	✓
K Q o	\$1.17	-\$2.76	\$0.50	✓
K J o	-\$0.52	-\$3.88	\$0.39	✓
K T o	-\$1.28	-\$4.51	\$0.29	✓
K 9 o	-\$1.91	-\$2.95	\$0.18	✓
K 8 o	-\$2.04	-\$3.34	\$0.12	✓
K 7 o	-\$2.09	-\$3.05	\$0.10	✓
K 6 o	-\$2.14	-\$2.66	\$0.08	✓
K 5 o	-\$2.12	-\$2.90	\$0.08	✓
K 4 o	-\$2.15	-\$2.58	\$0.07	✓
K 3 o	-\$2.12	-\$2.64	\$0.06	✓
K 2 o	-\$2.19	-\$2.64	\$0.06	✓
K A o	\$17.86	\$13.42	\$0.86	✓
Q J o	-\$1.24	-\$4.92	\$0.33	✓
Q T o	-\$1.52	-\$4.63	\$0.27	✓
Q 9 o	-\$1.89	-\$3.41	\$0.16	✓
Q 8 o	-\$2.11	-\$3.20	\$0.10	✓
Q 7 o	-\$2.17	-\$2.74	\$0.08	✓
Q 6 o	-\$2.15	-\$2.37	\$0.06	✓
Q 5 o	-\$2.15	-\$2.66	\$0.06	✓
Q 4 o	-\$2.17	-\$2.43	\$0.05	✓
Q 3 o	-\$2.21	-\$2.38	\$0.05	✓
Q 2 o	-\$2.19	-\$2.38	\$0.05	✓
Q A o	\$7.25	\$3.38	\$0.68	✓
J T o	-\$1.77	-\$4.70	\$0.28	✓
J 9 o	-\$2.03	-\$4.44	\$0.18	✓
J 8 o	-\$2.13	-\$3.36	\$0.12	✓
J 7 o	-\$2.15	-\$2.71	\$0.08	✓
J 6 o	-\$2.17	-\$2.65	\$0.06	✓
J 5 o	-\$2.20	-\$2.39	\$0.05	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
J 4 o	-\$2.20	-\$2.42	\$0.05	✓
J 3 o	-\$2.18	-\$2.55	\$0.05	✓
J 2 o	-\$2.18	-\$2.43	\$0.04	✓
J A o	\$2.67	-\$0.40	\$0.52	✓
T 9 o	-\$2.04	-\$3.86	\$0.21	✓
T 8 o	-\$2.19	-\$2.53	\$0.13	✓
T 7 o	-\$2.18	-\$3.05	\$0.09	✓
T 6 o	-\$2.19	-\$2.69	\$0.06	✓
T 5 o	-\$2.21	-\$2.52	\$0.05	✓
T 4 o	-\$2.17	-\$2.45	\$0.04	✓
T 3 o	-\$2.20	-\$2.26	\$0.04	
T 2 o	-\$2.19	-\$2.27	\$0.04	✓
T A o	\$0.56	-\$3.96	\$0.40	✓
9 8 o	-\$2.15	-\$3.74	\$0.18	✓
9 7 o	-\$2.18	-\$2.92	\$0.11	✓
9 6 o	-\$2.19	-\$2.90	\$0.07	✓
9 5 o	-\$2.23	-\$2.48	\$0.05	✓
9 4 o	-\$2.21	-\$2.49	\$0.04	✓
9 3 o	-\$2.19	-\$2.46	\$0.04	✓
9 2 o	-\$2.21	-\$2.33	\$0.04	✓
9 A o	-\$0.97	-\$3.87	\$0.27	✓
8 7 o	-\$2.14	-\$2.99	\$0.15	✓
8 6 o	-\$2.18	-\$2.51	\$0.10	✓
8 5 o	-\$2.19	-\$2.68	\$0.07	✓
8 4 o	-\$2.22	-\$2.38	\$0.05	✓

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
8 3 o	-\$2.25	-\$2.37	\$0.04	✓
8 2 o	-\$2.24	-\$2.31	\$0.03	✓
8 A o	-\$1.58	-\$4.91	\$0.23	✓
7 6 o	-\$2.13	-\$3.70	\$0.14	✓
7 5 o	-\$2.20	-\$2.67	\$0.09	✓
7 4 o	-\$2.22	-\$2.36	\$0.06	✓
7 3 o	-\$2.23	-\$2.24	\$0.04	
7 2 o	-\$2.25	-\$2.57	\$0.03	✓
7 A o	-\$1.69	-\$3.96	\$0.20	✓
6 5 o	-\$2.20	-\$3.79	\$0.12	✓
6 4 o	-\$2.21	-\$2.58	\$0.08	✓
6 3 o	-\$2.24	-\$2.47	\$0.06	✓
6 2 o	-\$2.23	-\$2.47	\$0.04	✓
6 A o	-\$1.93	-\$4.58	\$0.17	✓
5 4 o	-\$2.27	-\$2.87	\$0.10	✓
5 3 o	-\$2.20	-\$2.85	\$0.06	✓
5 2 o	-\$2.25	-\$2.44	\$0.05	✓
5 A o	-\$1.81	-\$3.67	\$0.18	✓
4 3 o	-\$2.24	-\$3.05	\$0.07	✓
4 2 o	-\$2.25	-\$2.52	\$0.05	✓
4 A o	-\$1.82	-\$4.16	\$0.16	✓
3 2 o	-\$2.26	-\$2.63	\$0.06	✓
3 A o	-\$2.02	-\$3.57	\$0.15	✓
2 A o	-\$2.05	-\$3.34	\$0.14	✓

of 169 starting hands in which players with *better* results on all other hands have statistically significant higher win rates: 144# of 169 starting hands in which players with *worse* results on all other hands have statistically significant higher win rates: 0

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Figure 22: Win rates for players with better and worse results on all other hands for each of 169 possible starting hands, for \$10/\$20 stakes

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
A A	\$202.55	\$194.14	\$8.61	
2 2	\$0.19	-\$6.74	\$3.38	✓
3 3	\$0.08	\$0.25	\$3.47	
4 4	\$0.92	-\$2.77	\$3.56	
5 5	\$3.24	-\$3.36	\$3.84	
6 6	\$5.03	-\$3.14	\$4.00	✓
7 7	\$9.04	-\$1.60	\$4.60	✓
8 8	\$15.06	\$5.91	\$5.03	
9 9	\$23.08	\$16.37	\$5.64	
T T	\$38.18	\$32.00	\$6.48	
J J	\$51.01	\$38.59	\$7.43	
Q Q	\$90.62	\$97.64	\$8.41	
K K	\$136.56	\$147.53	\$8.67	
K Q s	\$11.95	\$6.68	\$5.88	
K J s	\$10.19	\$8.21	\$5.37	
K T s	\$4.55	-\$6.24	\$4.78	✓
K 9 s	\$0.27	-\$4.64	\$3.78	
K 8 s	-\$2.70	-\$5.73	\$2.69	
K 7 s	-\$1.73	\$0.25	\$2.72	
K 6 s	-\$4.20	-\$4.41	\$2.28	
K 5 s	-\$1.83	-\$4.26	\$2.35	
K 4 s	-\$3.87	-\$1.97	\$2.37	
K 3 s	-\$5.94	-\$4.57	\$1.78	
K 2 s	-\$4.16	-\$9.17	\$1.99	✓
K A s	\$53.87	\$45.90	\$8.22	
Q J s	\$4.05	-\$1.70	\$5.17	
Q T s	\$5.55	\$1.65	\$4.63	
Q 9 s	-\$3.08	\$1.26	\$3.73	
Q 8 s	-\$2.57	-\$5.46	\$2.93	
Q 7 s	-\$4.83	-\$7.87	\$2.09	

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
Q 6 s	-\$4.41	-\$9.32	\$1.81	✓
Q 5 s	-\$4.09	-\$5.95	\$1.66	
Q 4 s	-\$3.91	-\$5.82	\$1.49	
Q 3 s	-\$2.99	-\$3.91	\$1.45	
Q 2 s	-\$4.32	-\$4.68	\$1.44	
Q A s	\$28.43	\$34.36	\$7.13	
J T s	\$2.72	\$0.63	\$5.14	
J 9 s	-\$1.56	-\$0.14	\$4.28	
J 8 s	-\$3.44	\$0.94	\$3.24	
J 7 s	-\$3.33	-\$8.49	\$2.41	✓
J 6 s	-\$3.75	-\$6.92	\$1.69	
J 5 s	-\$3.50	-\$6.66	\$1.57	✓
J 4 s	-\$4.71	-\$5.77	\$1.37	
J 3 s	-\$3.80	-\$5.76	\$1.26	
J 2 s	-\$4.59	-\$6.47	\$1.02	
J A s	\$13.45	\$11.72	\$5.88	
T 9 s	\$0.38	\$1.16	\$5.04	
T 8 s	-\$0.27	-\$1.99	\$3.80	
T 7 s	-\$3.60	-\$4.33	\$2.92	
T 6 s	-\$4.46	-\$6.66	\$1.80	
T 5 s	-\$4.16	-\$7.03	\$1.34	✓
T 4 s	-\$4.42	-\$6.85	\$1.10	✓
T 3 s	-\$4.43	-\$6.04	\$1.13	
T 2 s	-\$4.09	-\$3.61	\$0.97	
T A s	\$11.38	\$5.65	\$5.27	
9 8 s	\$0.19	-\$3.17	\$4.79	
9 7 s	\$1.22	-\$5.20	\$3.77	
9 6 s	-\$3.47	-\$10.14	\$2.41	✓
9 5 s	-\$4.14	-\$6.35	\$1.53	
9 4 s	-\$4.33	-\$4.21	\$1.06	

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
9 3 s	-\$4.26	-\$3.45	\$0.93	
9 2 s	-\$4.44	-\$5.36	\$0.71	
9 A s	\$5.04	\$1.94	\$4.43	
8 7 s	-\$4.72	-\$9.04	\$4.87	
8 6 s	-\$4.25	-\$1.63	\$3.36	
8 5 s	-\$4.31	-\$7.77	\$2.37	
8 4 s	-\$4.22	-\$4.12	\$1.47	
8 3 s	-\$5.65	-\$5.90	\$1.02	
8 2 s	-\$4.68	-\$5.74	\$0.71	
8 A s	\$2.00	\$4.92	\$4.13	
7 6 s	\$0.79	-\$6.24	\$4.44	
7 5 s	-\$3.67	\$0.60	\$3.46	
7 4 s	-\$4.64	-\$7.09	\$2.06	
7 3 s	-\$5.80	-\$7.22	\$1.15	
7 2 s	-\$4.41	-\$7.16	\$0.89	✓
7 A s	-\$0.20	-\$6.33	\$4.00	
6 5 s	\$0.02	-\$5.51	\$4.04	
6 4 s	-\$2.73	-\$5.41	\$3.05	
6 3 s	-\$3.51	-\$4.91	\$2.28	
6 2 s	-\$5.10	-\$4.24	\$1.20	
6 A s	-\$2.43	-\$2.62	\$3.77	
5 4 s	-\$0.97	-\$7.55	\$3.44	
5 3 s	-\$4.41	-\$6.31	\$2.52	
5 2 s	-\$4.27	-\$1.89	\$1.52	
5 A s	\$1.37	-\$3.24	\$3.88	
4 3 s	-\$5.98	-\$2.53	\$2.84	
4 2 s	-\$5.53	-\$4.26	\$1.68	
4 A s	-\$3.99	-\$3.64	\$4.09	
3 2 s	-\$4.99	-\$5.39	\$1.50	
3 A s	\$1.93	-\$0.74	\$3.81	

Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
2 A s	-\$0.45	\$2.45	\$3.57	
K Q o	\$2.45	-\$3.32	\$2.62	✓
K J o	-\$0.62	-\$8.24	\$2.01	✓
K T o	-\$2.28	-\$4.04	\$1.53	
K 9 o	-\$4.20	-\$6.87	\$0.92	✓
K 8 o	-\$3.90	-\$5.71	\$0.60	✓
K 7 o	-\$4.28	-\$5.45	\$0.49	✓
K 6 o	-\$4.69	-\$5.02	\$0.45	
K 5 o	-\$4.38	-\$3.33	\$0.38	✓
K 4 o	-\$4.39	-\$4.39	\$0.37	
K 3 o	-\$4.39	-\$4.50	\$0.30	
K 2 o	-\$4.38	-\$5.39	\$0.32	✓
K A o	\$36.25	\$28.32	\$4.53	
Q J o	-\$2.03	-\$4.98	\$1.71	
Q T o	-\$3.91	-\$7.22	\$1.46	✓
Q 9 o	-\$3.86	-\$5.00	\$0.85	
Q 8 o	-\$4.77	-\$5.38	\$0.57	
Q 7 o	-\$4.46	-\$5.44	\$0.35	✓
Q 6 o	-\$4.32	-\$5.08	\$0.32	✓
Q 5 o	-\$4.51	-\$5.15	\$0.31	✓
Q 4 o	-\$4.62	-\$4.97	\$0.26	
Q 3 o	-\$4.44	-\$4.47	\$0.23	
Q 2 o	-\$4.63	-\$4.81	\$0.24	
Q A o	\$13.08	\$7.59	\$3.55	
J T o	-\$2.78	-\$6.02	\$1.44	✓
J 9 o	-\$4.86	-\$8.94	\$0.91	✓
J 8 o	-\$4.10	-\$6.68	\$0.60	✓
J 7 o	-\$4.50	-\$5.68	\$0.42	✓
J 6 o	-\$4.49	-\$4.30	\$0.22	
J 5 o	-\$4.41	-\$5.13	\$0.25	✓

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Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
J 4 o	-\$4.43	-\$5.13	\$0.23	✓
J 3 o	-\$4.53	-\$4.84	\$0.25	
J 2 o	-\$4.63	-\$4.96	\$0.22	
J A o	\$4.18	\$0.12	\$2.66	
T 9 o	-\$3.71	-\$9.44	\$1.17	✓
T 8 o	-\$4.66	-\$7.95	\$0.67	✓
T 7 o	-\$4.29	-\$6.51	\$0.43	✓
T 6 o	-\$4.41	-\$5.83	\$0.34	✓
T 5 o	-\$4.46	-\$5.01	\$0.21	✓
T 4 o	-\$4.54	-\$4.22	\$0.21	
T 3 o	-\$4.51	-\$4.29	\$0.17	
T 2 o	-\$4.59	-\$4.77	\$0.17	
T A o	\$1.75	-\$4.29	\$2.07	✓
9 8 o	-\$4.42	-\$7.01	\$0.90	✓
9 7 o	-\$4.32	-\$5.54	\$0.70	
9 6 o	-\$4.67	-\$6.37	\$0.41	✓
9 5 o	-\$4.59	-\$4.84	\$0.25	
9 4 o	-\$4.54	-\$4.95	\$0.20	
9 3 o	-\$4.59	-\$4.77	\$0.17	
9 2 o	-\$4.52	-\$4.82	\$0.15	
9 A o	-\$2.16	-\$3.96	\$1.37	
8 7 o	-\$4.60	-\$6.35	\$0.80	✓
8 6 o	-\$4.32	-\$5.93	\$0.52	✓
8 5 o	-\$4.44	-\$5.34	\$0.33	✓
8 4 o	-\$4.44	-\$4.50	\$0.23	

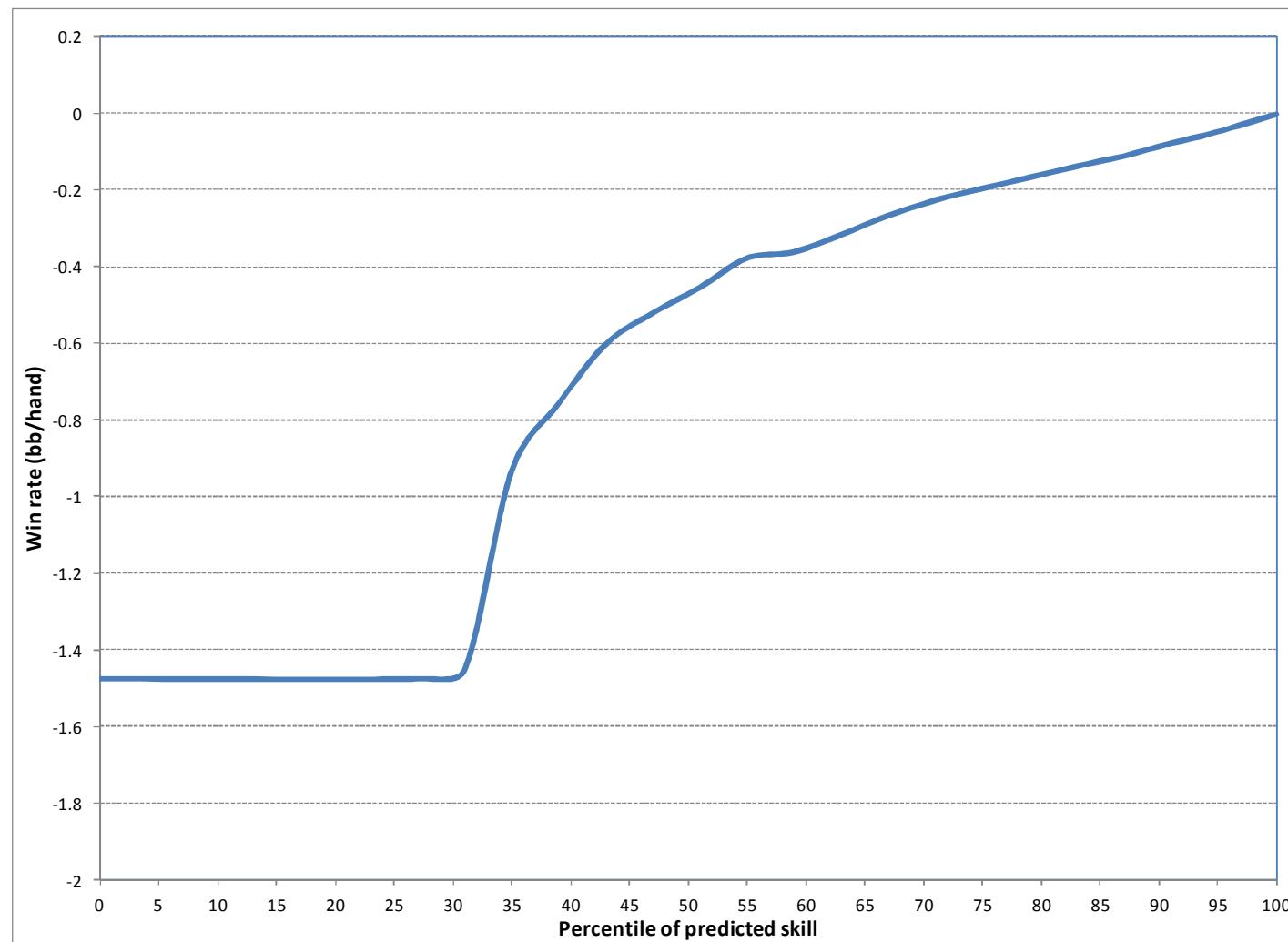
Starting hand	Win rates for players with		Standard error	Statistically significant difference?
	better results on all other hands	worse results on all other hands		
8 3 o	-\$4.45	-\$4.98	\$0.19	✓
8 2 o	-\$4.51	-\$4.85	\$0.18	
8 A o	-\$3.29	-\$9.82	\$1.17	✓
7 6 o	-\$5.01	-\$8.48	\$0.76	✓
7 5 o	-\$4.43	-\$5.14	\$0.50	
7 4 o	-\$4.61	-\$5.79	\$0.31	✓
7 3 o	-\$4.49	-\$4.76	\$0.18	
7 2 o	-\$4.53	-\$4.55	\$0.16	
7 A o	\$2.82	-\$5.46	\$0.98	✓
6 5 o	-\$4.66	-\$5.36	\$0.63	
6 4 o	-\$4.49	-\$5.68	\$0.36	✓
6 3 o	-\$4.61	-\$5.64	\$0.26	✓
6 2 o	-\$4.48	-\$4.29	\$0.23	
6 A o	-\$4.57	-\$6.46	\$0.84	✓
5 4 o	-\$4.71	-\$3.86	\$0.58	
5 3 o	-\$4.44	-\$4.84	\$0.38	
5 2 o	-\$4.45	-\$5.00	\$0.21	✓
5 A o	-\$3.82	-\$5.57	\$0.95	
4 3 o	-\$4.58	-\$4.88	\$0.38	
4 2 o	-\$4.63	-\$4.86	\$0.22	
4 A o	-\$4.06	-\$5.36	\$0.84	
3 2 o	-\$4.82	-\$6.40	\$0.32	✓
3 A o	-\$4.27	-\$6.30	\$0.75	✓
2 A o	-\$4.10	-\$6.20	\$0.67	✓

of 169 starting hands in which players with *better* results on all other hands have statistically significant higher win rates: 51# of 169 starting hands in which players with *worse* results on all other hands have statistically significant higher win rates: 1

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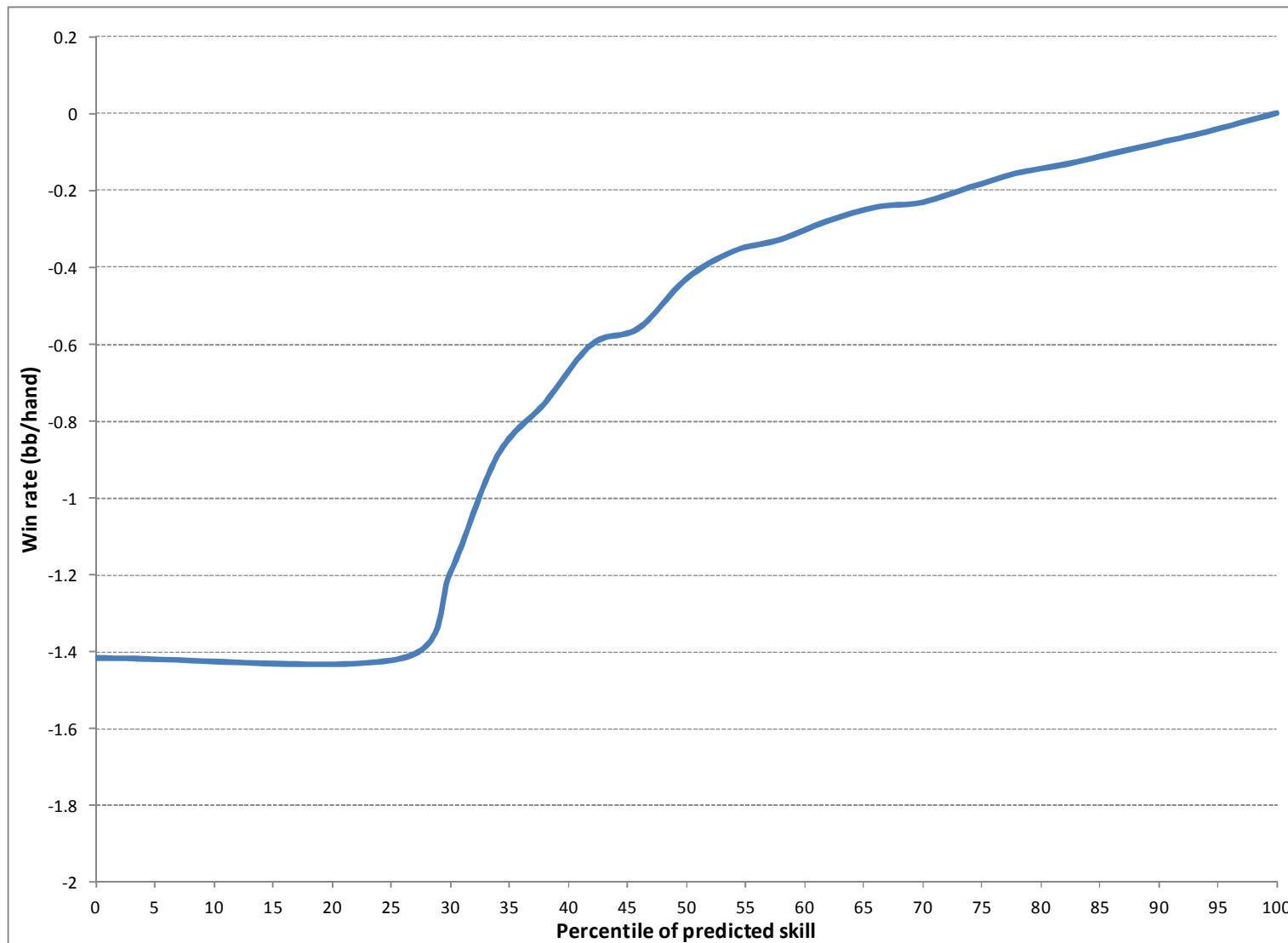
IV.C. Average win rates for players of different predicted skill

Figure 23: Average win rate for players of different predicted skill, for \$0.50/\$1.00 stakes players in the prediction group



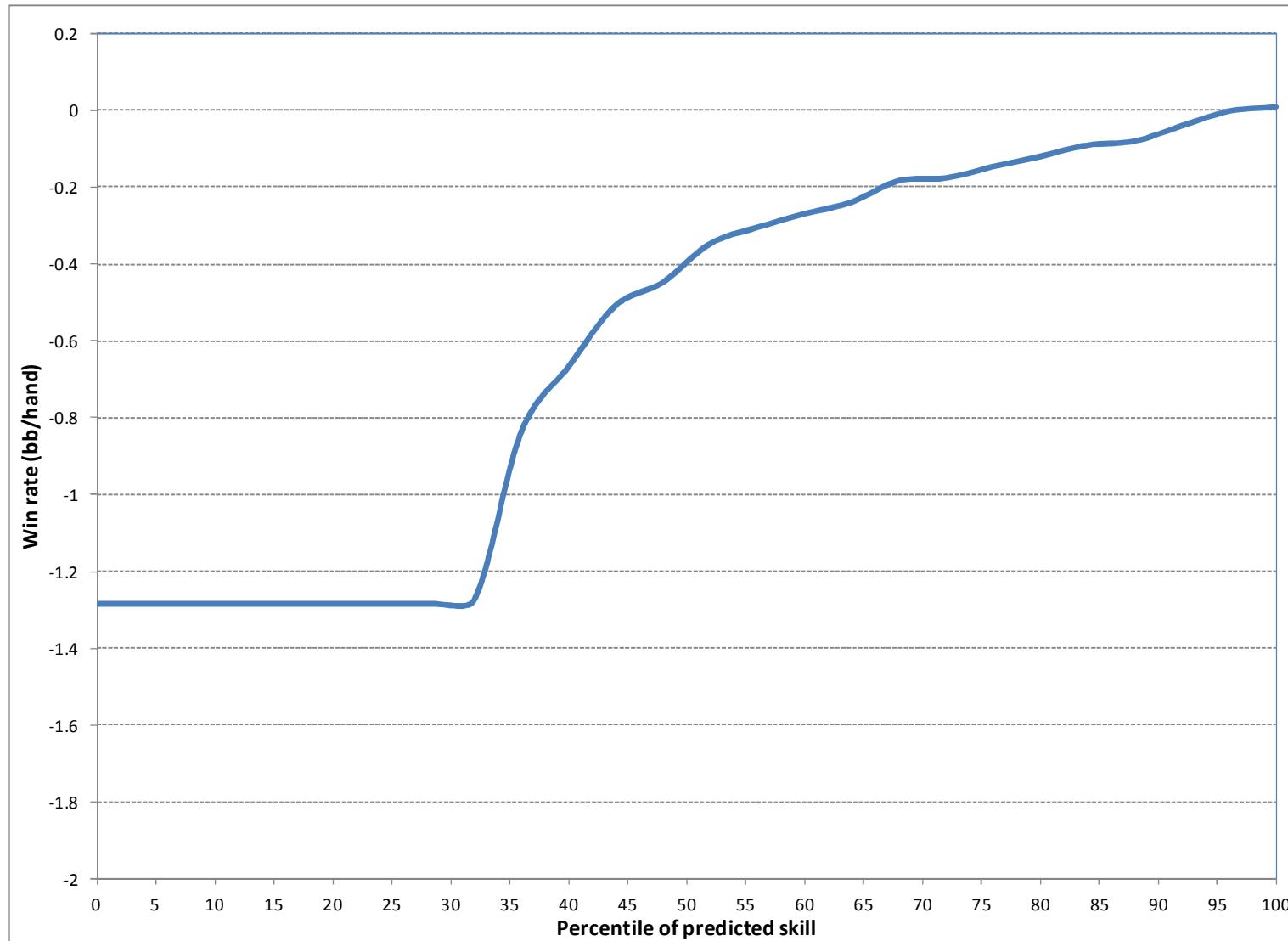
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Figure 24: Average win rate for players of different predicted skill, for \$1/\$2 stakes players in the prediction group



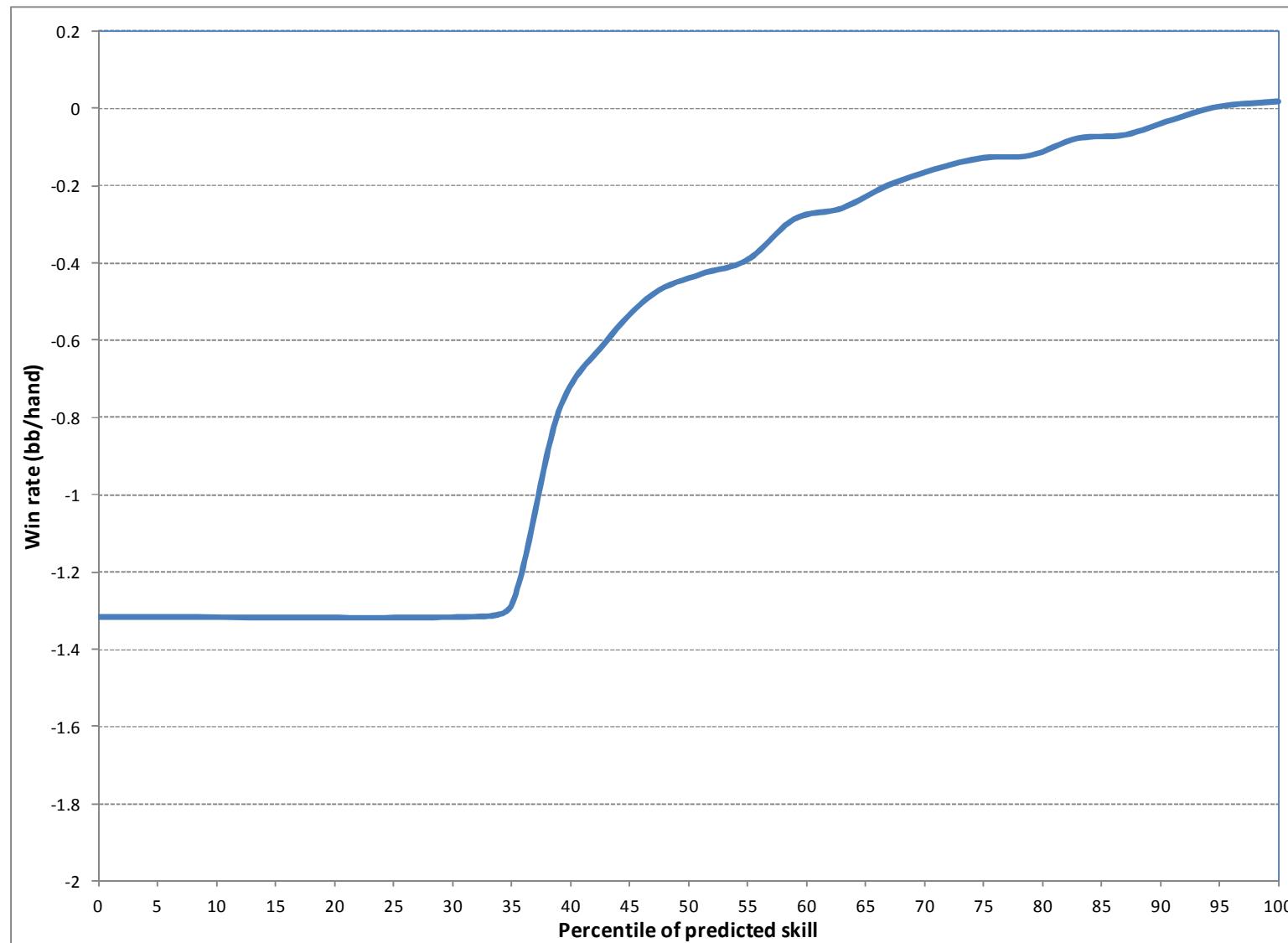
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Figure 25: Average win rate for players of different predicted skill, for \$2/\$4 stakes players in the prediction group



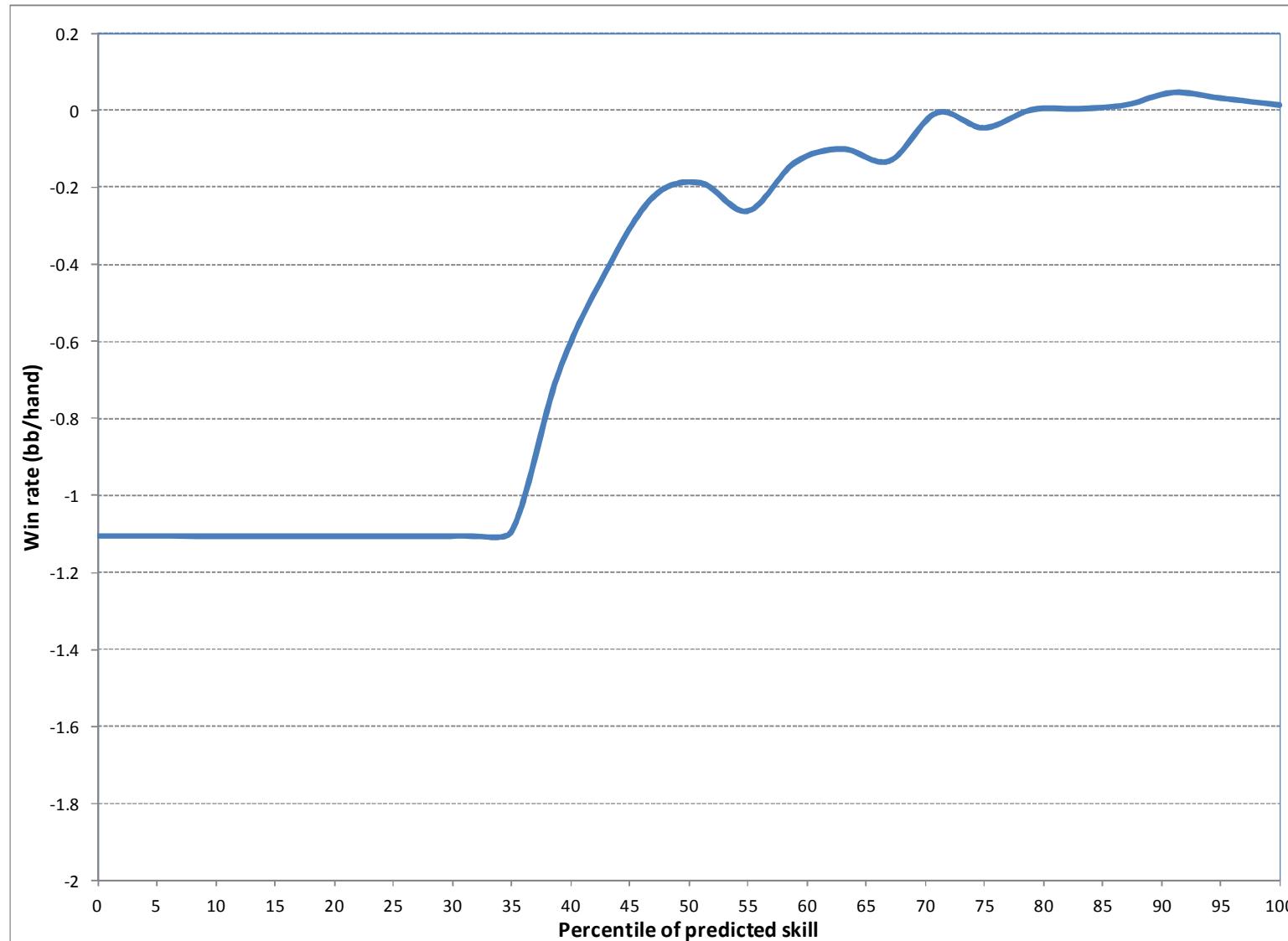
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Figure 26: Average win rate for players of different predicted skill, for \$3/\$6 stakes players in the prediction group



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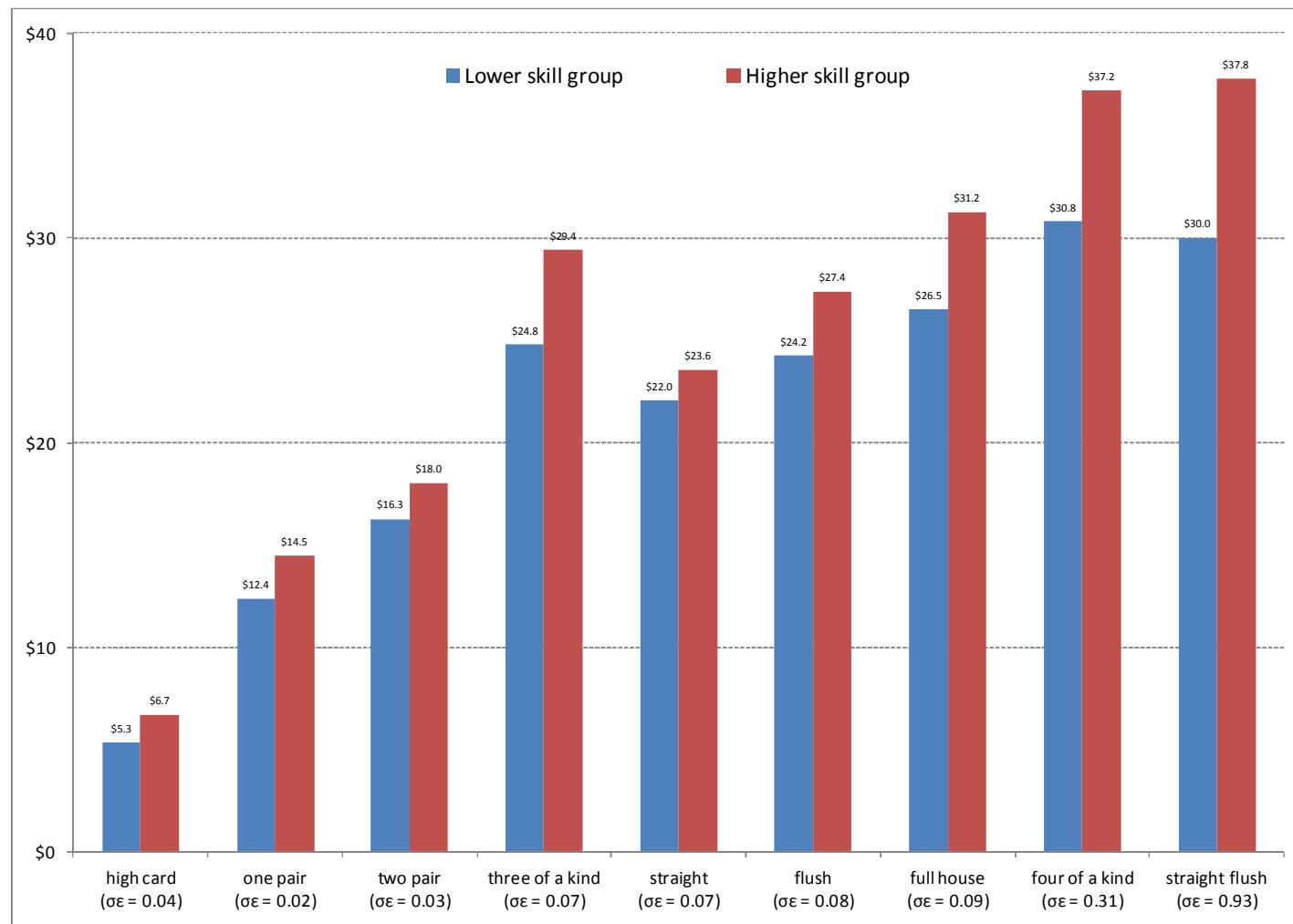
Figure 27: Average win rate for players of different predicted skill, for \$10/\$20 stakes players in the prediction group



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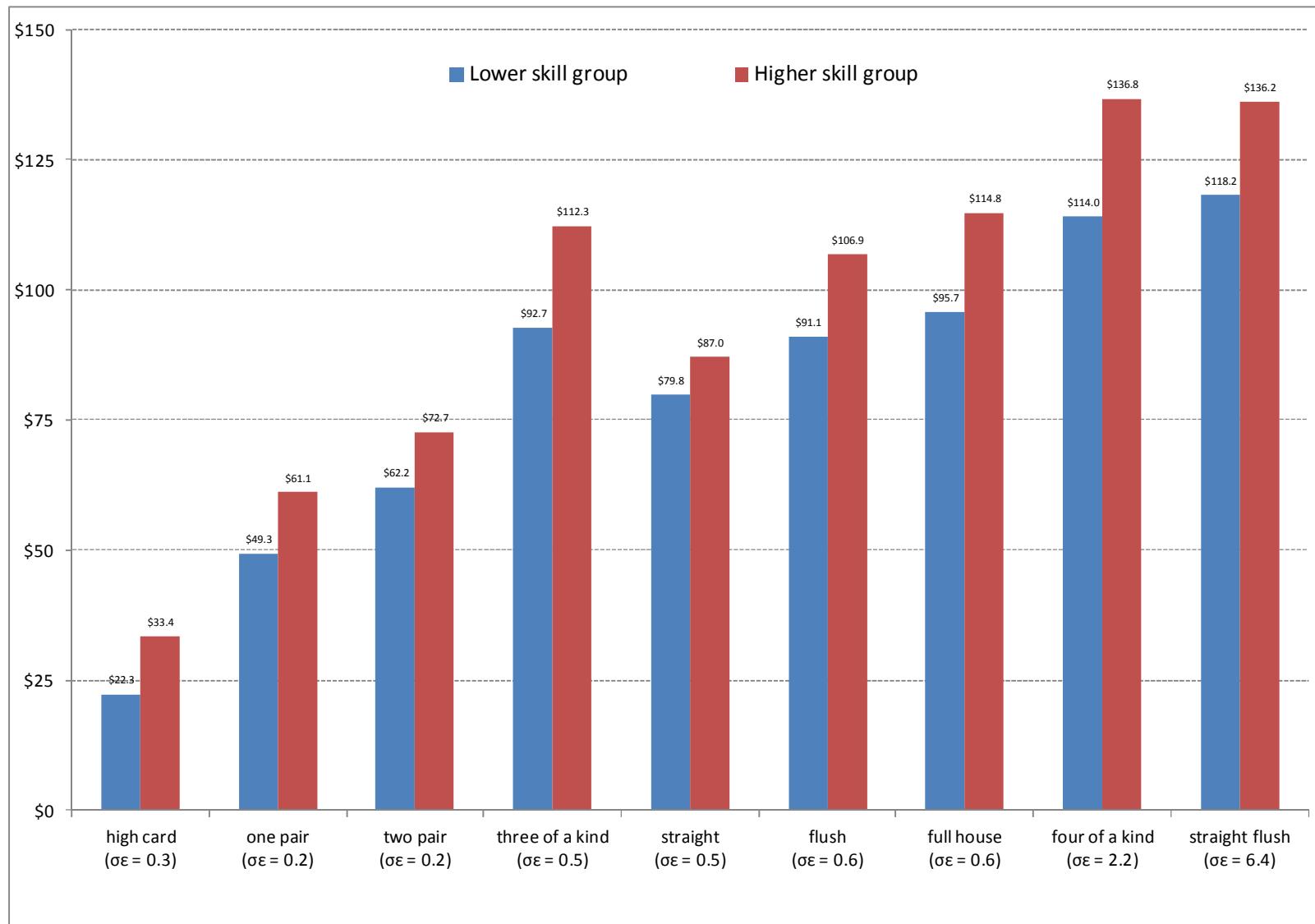
IV.D. Win rates for higher skilled and lower skilled players with the same hand category

Figure 28: Win rates for higher skilled and lower skilled players with the same hand category, \$0.50/\$1.00 stakes



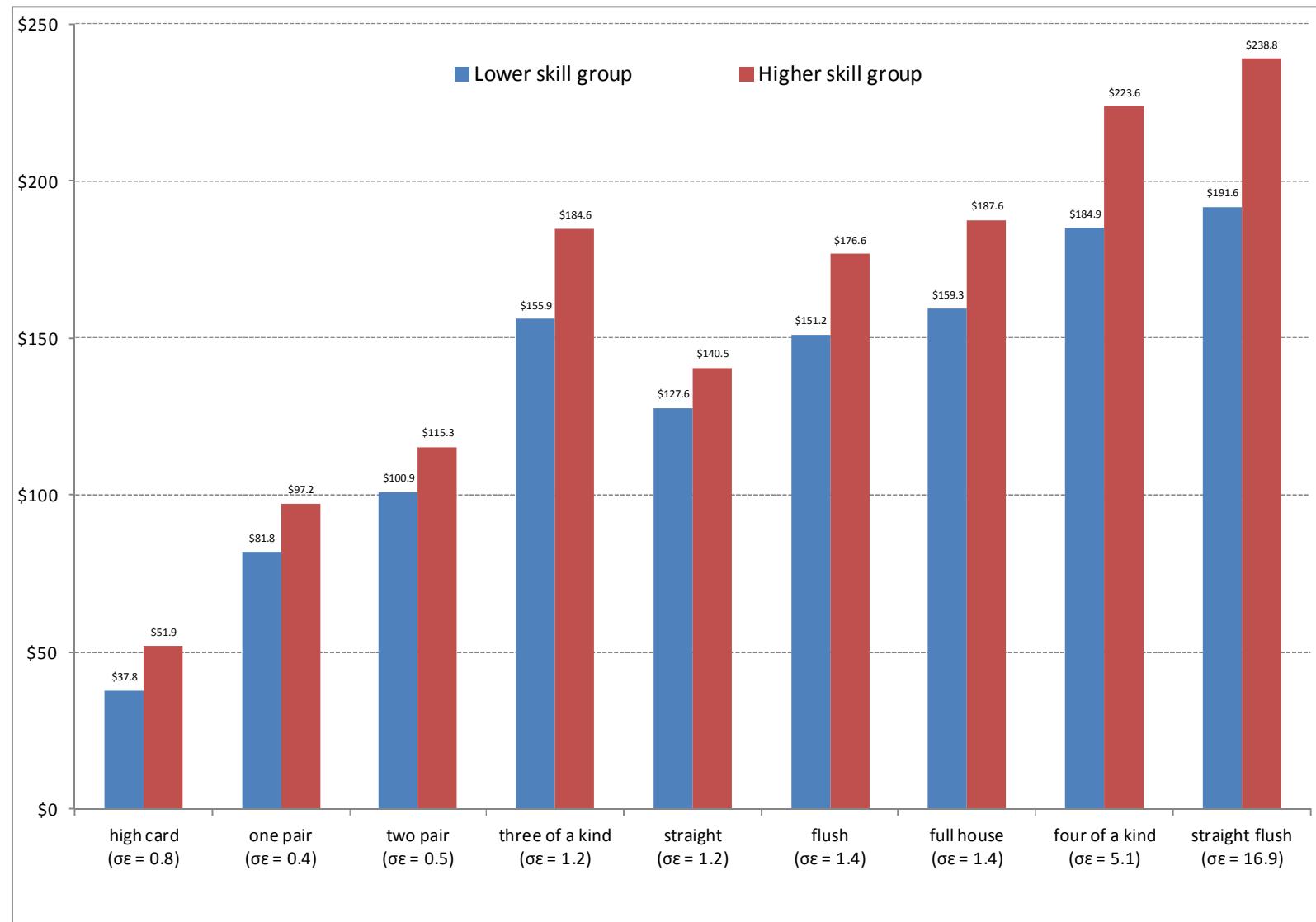
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Figure 29: Win rates for higher skilled and lower skilled players with the same hand category, \$2/\$4 stakes



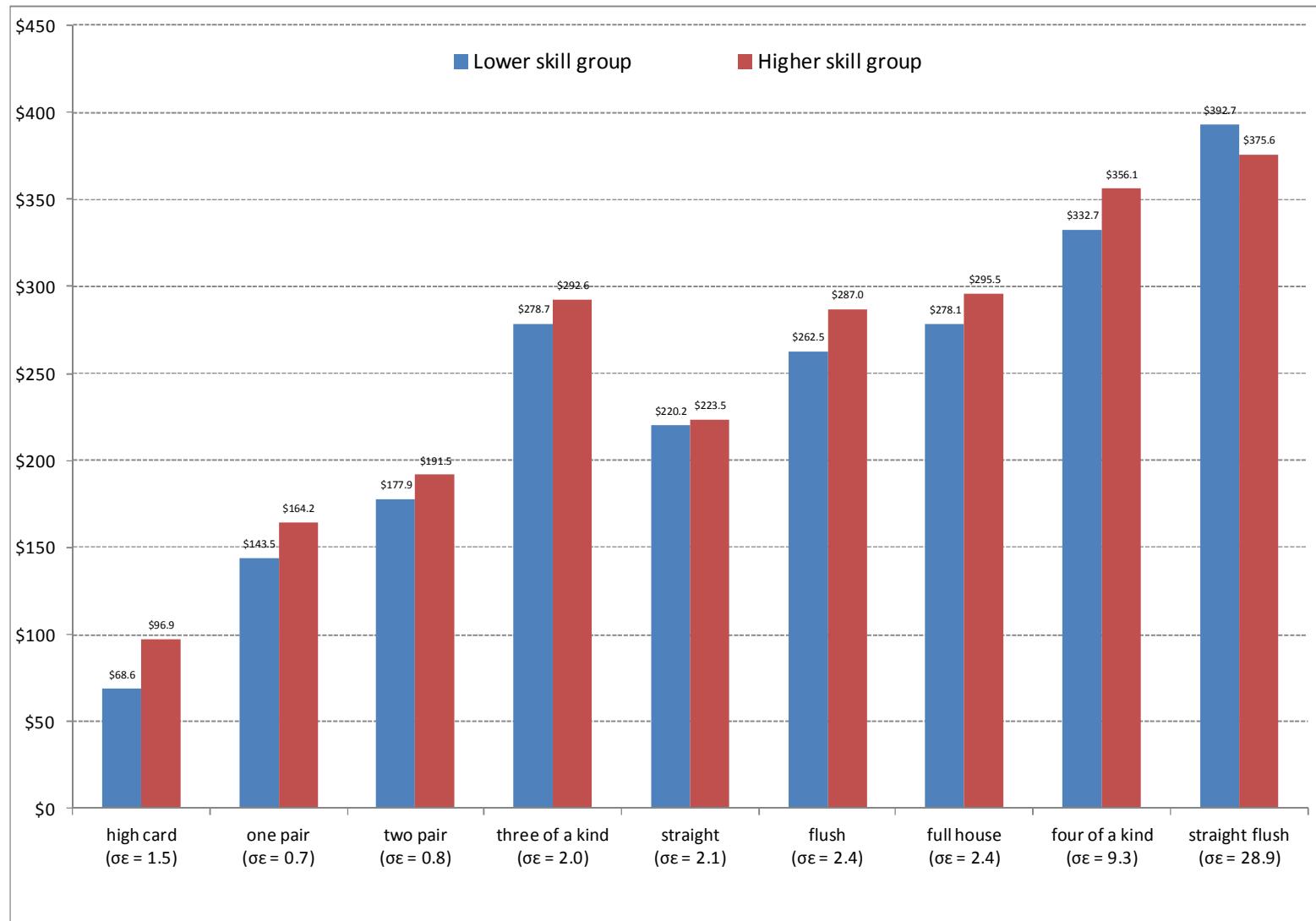
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Figure 30: Win rates for higher skilled and lower skilled players with the same hand category, \$3/\$6 stakes



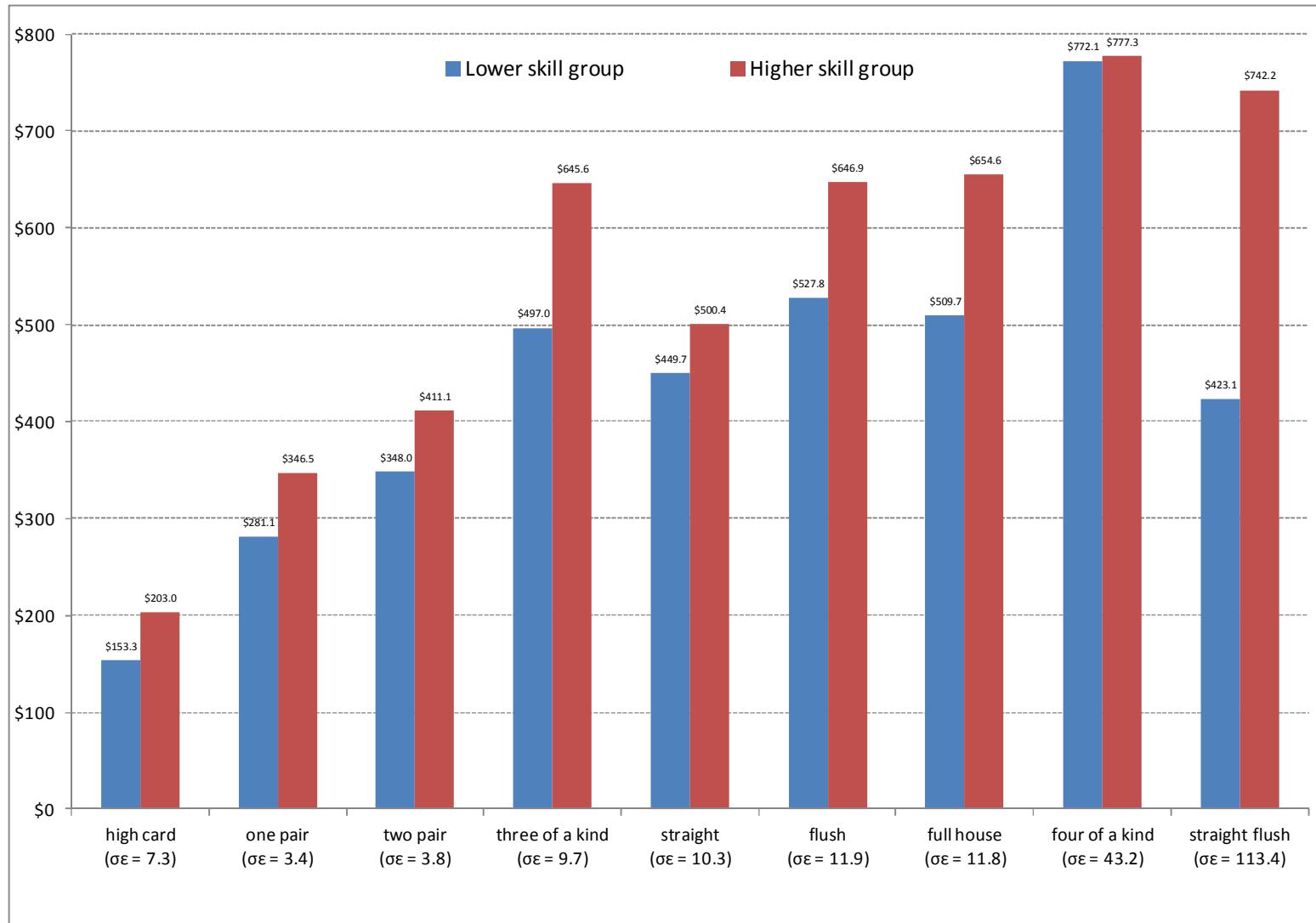
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Figure 31: Win rates for higher skilled and lower skilled players with the same hand category, \$5/\$10 stakes



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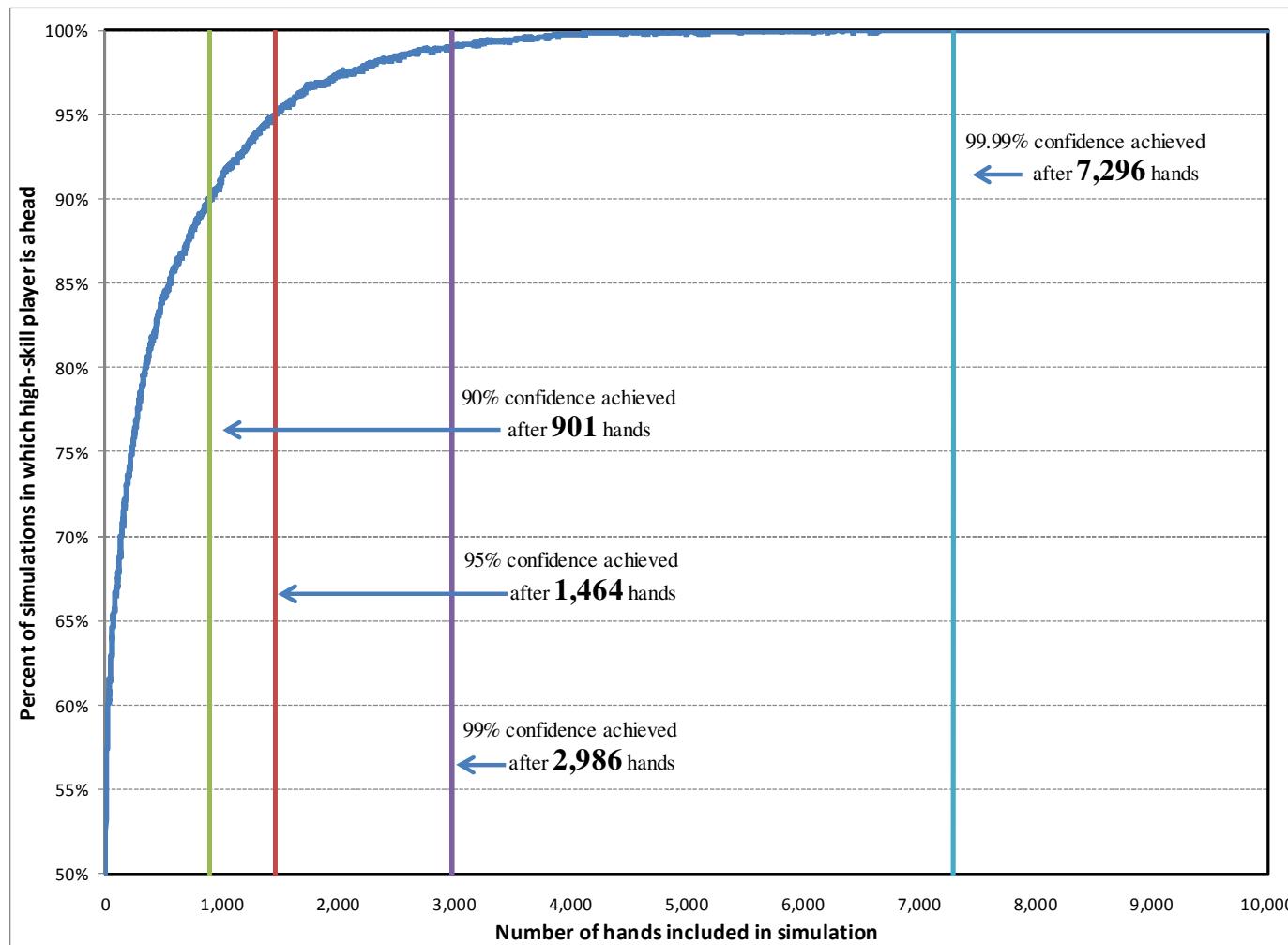
Figure 32: Win rates for higher skilled and lower skilled players with the same hand category, \$10/\$20 stakes



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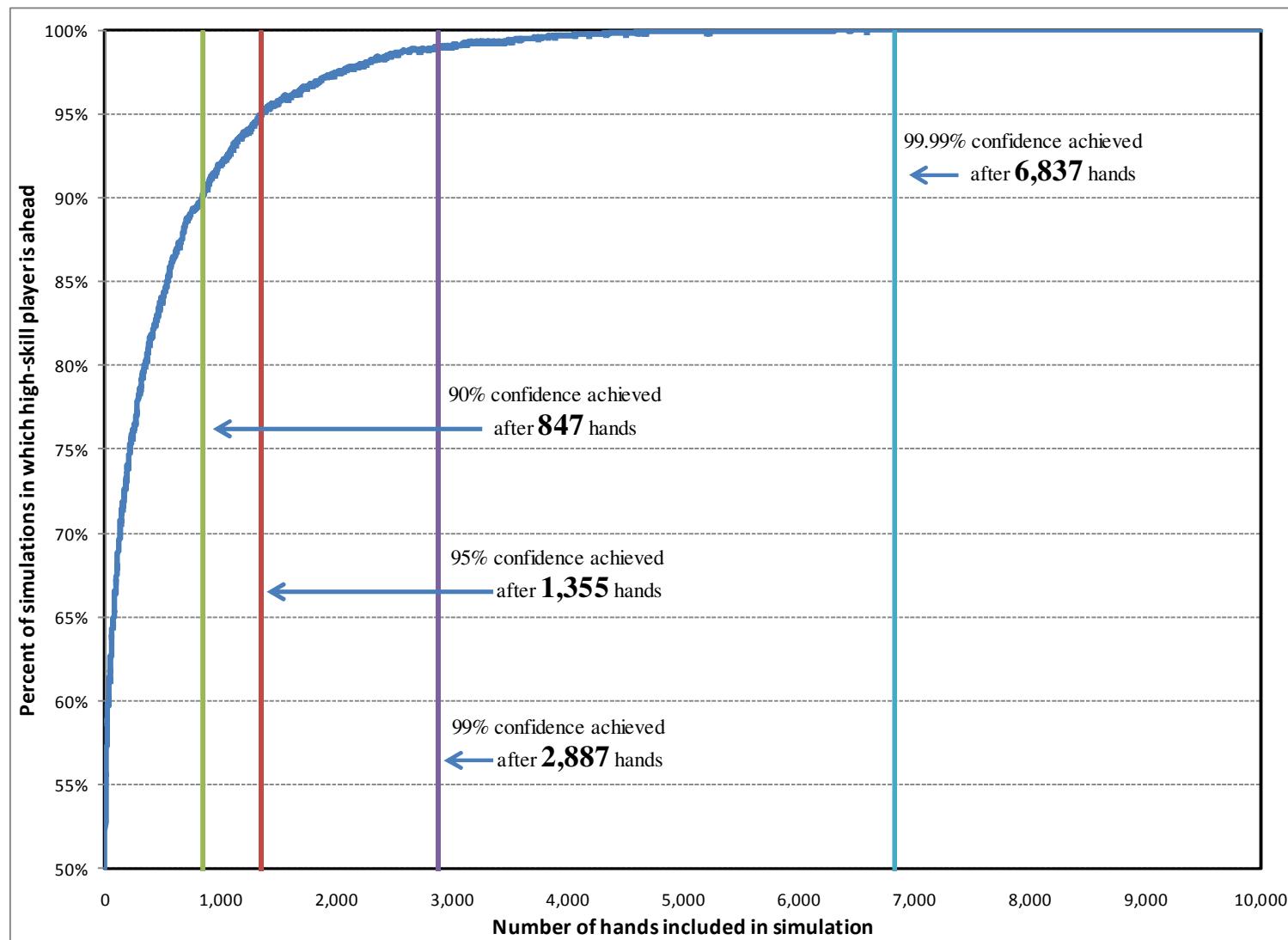
IV.E. Percentage of the time a higher skilled player would predominate over a lower skilled player

Figure 33: Percentage of the time a higher skilled player (top 50% of skill) would predominate over a lower skilled player (bottom 50% of skill) after any given number of hands at \$0.50/\$1.00 stakes



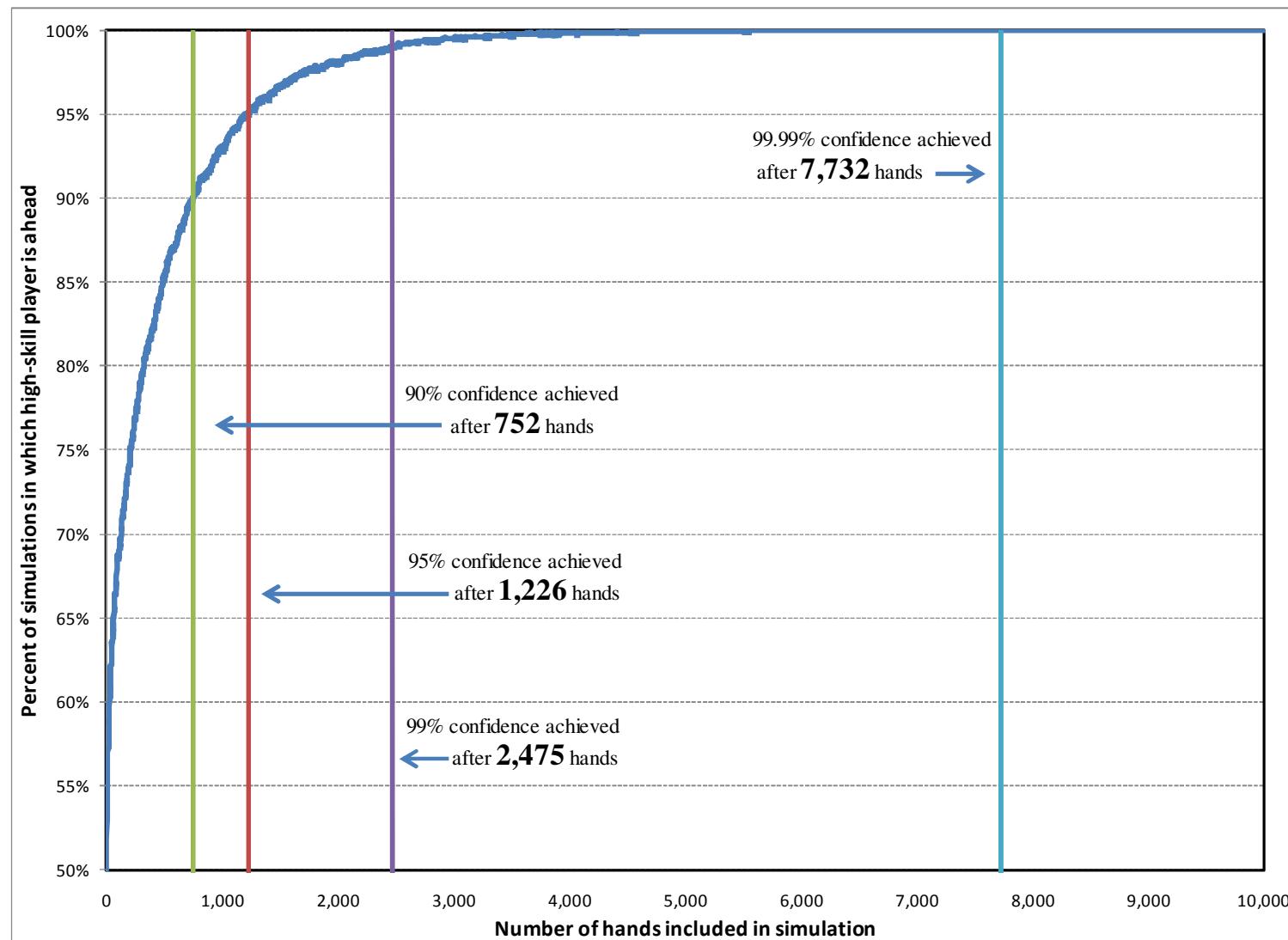
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Figure 34: Percentage of the time a higher skilled player (top 50% of skill) would predominate over a lower skilled player (bottom 50% of skill) after any given number of hands at \$2/\$4 stakes



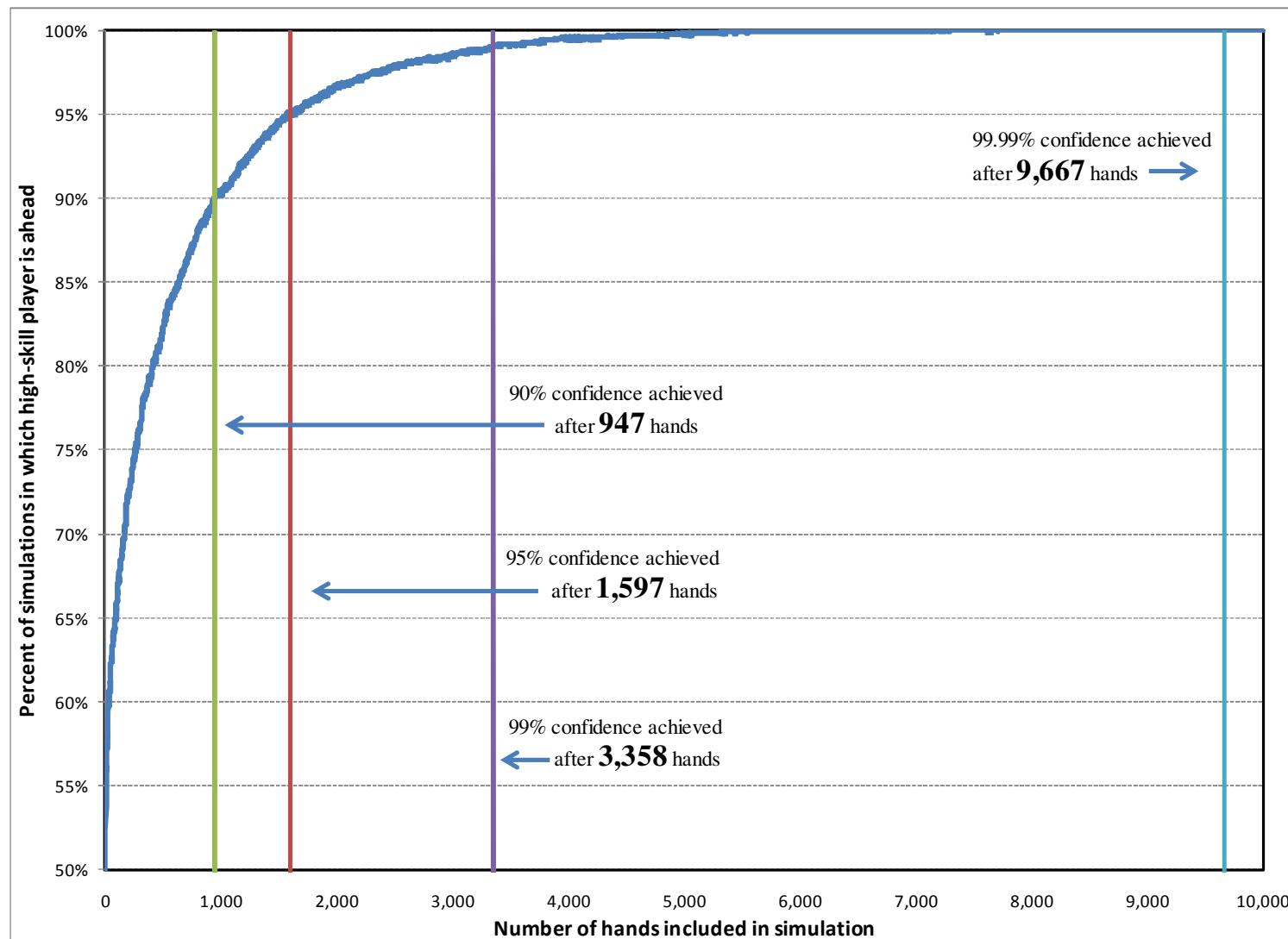
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Figure 35: Percentage of the time a higher skilled player (top 50% of skill) would predominate over a lower skilled player (bottom 50% of skill) after any given number of hands at \$3/\$6 stakes



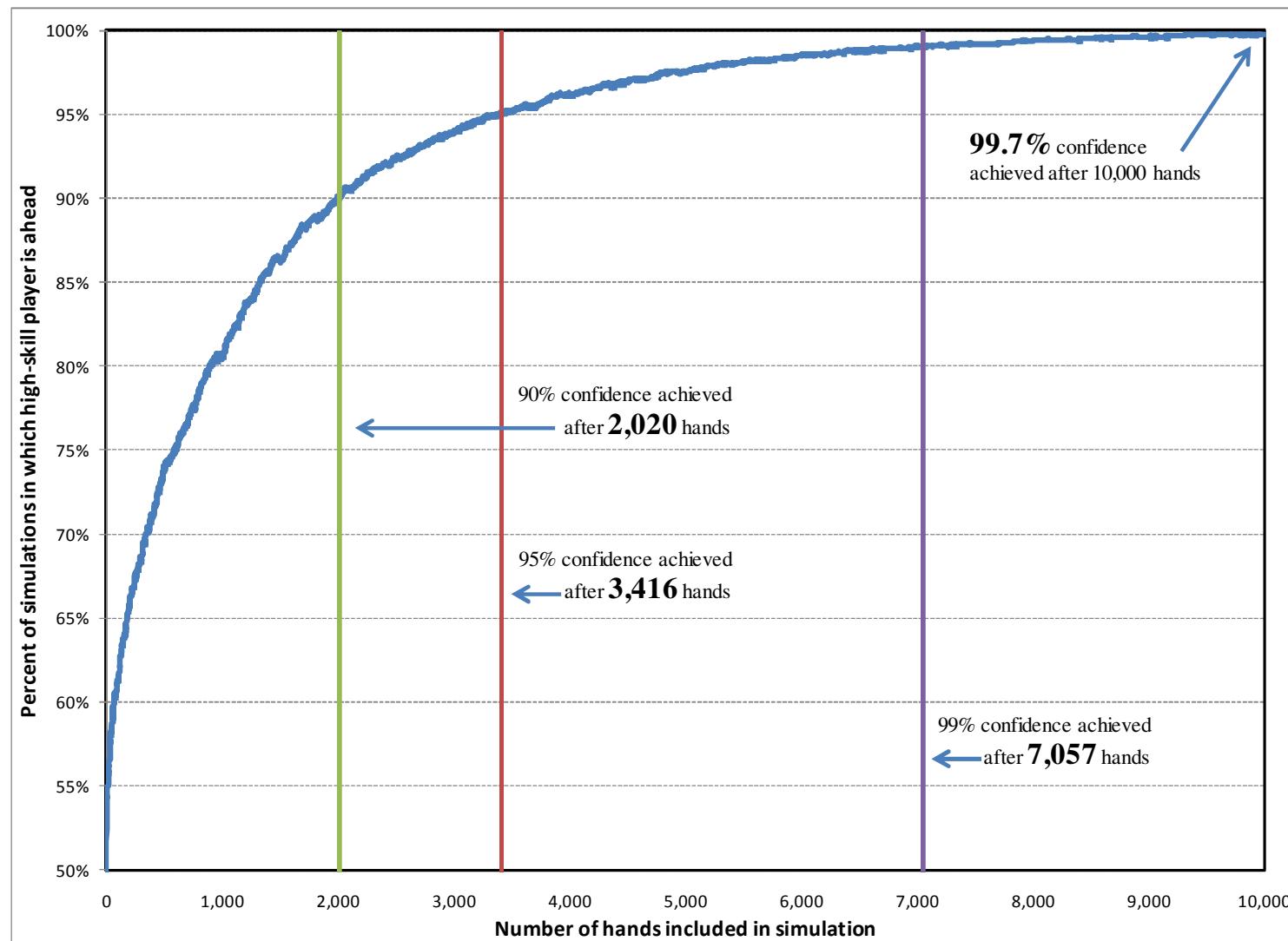
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Figure 36: Percentage of the time a higher skilled player (top 50% of skill) would predominate over a lower skilled player (bottom 50% of skill) after any given number of hands at \$5/\$10 stakes



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Figure 37: Percentage of the time a higher skilled player (top 50% of skill) would predominate over a lower skilled player (bottom 50% of skill) after any given number of hands at \$10/\$20 stakes



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IV.F. Percentage of simulations in which the lower skilled player catches up to the higher skilled player

Figure 38: Likelihood a trailing, lower skilled player catches up to a higher skilled player after an additional number of hands equivalent to the number of hands required to reach 95% and 99% confidence

Stakes	95%	99%
\$0.50/\$1.00	43 out of 9500	2 out of 9900
\$1/\$2	34 out of 9500	0 out of 9900
\$2/\$4	46 out of 9500	2 out of 9900
\$3/\$6	44 out of 9500	5 out of 9900
\$5/\$10	40 out of 9500	1 out of 9900
\$10/\$20	45 out of 9500	1 out of 9900

IV.G. Extensions of the conclusions to closely related games

- (79) I conducted my analyses on cash games played on 6-player maximum tables at stakes from \$0.50/\$1.00 through \$10/\$20 playing online No Limit Hold'em, also commonly called No Limit Texas Hold'em. Based on my experience as a poker player and as an econometrician, it is my opinion that this result, that poker is a game in which skill predominates over chance, extends directly to other closely related poker games. In particular, this result extends to poker games played on 9 and 10 player max tables. It also extends to all stakes, however much higher or lower than those that I analyzed. It also extends to poker games played with Limit and Pot-Limit betting rules. It extends to Omaha Hold'em and Omaha Hi/Lo Hold'em. It also extends to tournament play. It extends to all combinations and permutations of the above, so it extends, for example to Pot Limit Omaha tournaments. There are many other poker games and rules permutations to which the above conclusion also might be properly extended. I do not intend to imply, by drawing the line to include all of the above games, that any of these other permutations is not a game of skill, but only that I would need to conduct further analysis to address that question.
- (80) All of the elements of skill that are present in 6-player maximum No Limit Hold'em are also present in play on 9-player maximum and 10-player maximum tables. The extra three or four players somewhat increases the risk that other players may have unusually strong hands, since there are more players dealt cards in each hand. In addition, there are three or four more positions for which it is necessary to have slightly differentiated strategies. Overall, these games are so similar to 6-player

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maximum games that the conclusion that skill predominates over chance extends immediately to these other games.

- (81) Since poker is a game in which skill predominates over chance at stakes between \$0.50/\$1.00 and \$10/\$20, logic implies that it is a game in which skill predominates over chance at all stakes, since it is the same game being played. However, there are two important considerations about the stakes levels that need to be pointed out. At very low stakes there are many, many players playing unskillful strategies, presumably because so little is at risk that they do not mind losing. In this environment a skillful player will predominate easily and relatively quickly. At higher stakes, the players tend to be either better or richer or both. When the players against whom one competes are more skillful, it takes longer for the increased success of skillful players to manifest itself.
- (82) I have significant experience with alternatives to the No Limit betting rules, most notably with Pot Limit and Limit rules. Pot Limit is similar in many respects to No Limit, though there are different considerations that affect Pot Limit. For example, in Pot Limit, there is more scope to control the size of the pot by strategically manipulating bet sizes. Limit is arguably a simpler game than No Limit, and therefore may require less skill. Nonetheless, until the recent popularity of No Limit, most poker was played with Limit betting rules, and generations of professional poker players have made a living by dominating over lesser players playing Limit poker, including Limit Hold'em, demonstrating that Limit Hold'em is also a game in which skill predominates over chance.
- (83) Omaha and Omaha Hi/Lo are Hold'em variations in which four cards rather than two cards are dealt to each player. In Hi/Lo the winner also splits the pot with the player holding the best Low hand. Like Texas Hold'em, these games have five common cards and four betting rounds, and similar blind structures. The value of the hands, of course, are different, as are the odds of winning from various positions, but the strategic considerations are essentially the same, as is the scope for skill to prevail. Once again, countless poker professionals around the world have demonstrated that skill predominates over chance in these games by consistently predominating over less skillful adversaries for long periods of time. Tournament poker play involves players buying in to an event in which the winnings are distributed not on each hand, but rather based on how long a player survives over a continuous and sometimes long series of hands. Players generally cannot buy more chips if they lose their initial stake, or at least there is a limit to how often or when a player can buy more chips. The last player remaining with chips is the winner, and the last player eliminated gets second place, etc., back to the furthest position to receive a payoff. The World Series of Poker is the most famous example of this type of poker tournament. Many smaller versions are popular, including events in which just nine players compete at a single table until only one is left. Tournament play requires additional considerations that are different than cash games. For example, players must adjust their strategies for the number of players remaining. Stake sizes and the payoff schedule add strategic wrinkles that influence some choices. Based upon the fact that tournament play adds additional

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elements of skill and strategy on top of a game that I have already demonstrated is a game of skill, it is my opinion that skill predominates over chance in tournament poker.

- (84) It is my opinion that all combinations of the variations discussed above are also games in which skill predominates over chance, so this conclusion extends to Pot Limit Omaha tournaments, for example. There are many other poker games and rules permutations to which the above conclusion also might be properly extended. I do not intend to imply, by drawing the line to include all of the above games, that any of these other poker permutations is not a game of skill, but only that I would need to conduct further analysis or give further consideration to address that question.